FOCUSED REVIEW

Focused Review: Damping, Driving, and Resonance — Calculus-Based

Review the highest-leverage ideas about damping, driving, and resonance using the Calculus-Based treatment.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
15–20 minutes	Focused review	Greater confidence

After this focused review, you'll be able to…
analyze damped and driven oscillator equations, connect transient and steady-state solutions, and interpret amplitude and phase response.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 15–20 minutes

FR-02 • COURSE ALIGNMENT

Course Alignment

This Physics Sensei Unit Review supports MEC-U10 — Damping, Driving, and Resonance. Use it to reinforce concepts, prepare for homework, or review before a quiz or exam.

UNIT: MEC-U10 **TOPIC: Damping, Driving, and Resonance** **TREATMENT: Calculus-Based** **RESOURCE: Physics Sensei Unit Review**	**BEST USED** ✓ After reading the chapter ✓ Before starting homework ✓ Before a quiz or exam

Explore Physics Sensei →

Physics Sensei is an independent physics learning resource. This review uses original Physics Sensei organization, explanations, examples, and practice.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six focused stages in order. Each stage reinforces your understanding and prepares you for a final confidence check.

6 stages • Approximately 15–20 minutes

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a minute to reactivate what you already know. This quick warm-up will help you focus on the most important ideas before moving on.

WARM-UP 1

Key Ideas

Read the homogeneous equation.

For $mx'' + bx' + kx = 0$, what determines the roots of the trial solution $x = e^{rt}$?

WARM-UP 2

Common Mistakes

Separate transient and steady state.

In $mx'' + bx' + kx = F_0 \cos(\omega t)$, which part depends on initial conditions?

FR-04 • WARM-UP CHECK CONTINUED

WARM-UP 3

Quick Application

Interpret complex roots.

What do complex conjugate characteristic roots with negative real part imply physically?

Ready to strengthen your understanding?

You’ve refreshed the most important ideas. Now reinforce the concepts you’ll use most often.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's reinforce the most important concepts you'll need to solve problems with confidence. Focus on the ideas that appear most often in homework, quizzes, and exams.

KEY CONCEPT 1

Homogeneous Dynamics and Damping Regimes

For $mx'' + bx' + kx = 0$, the characteristic equation $mr^2 + br + k = 0$ has roots $r = [-b \pm \sqrt{(b^2 - 4mk)}]/(2m)$. The discriminant separates underdamped, critically damped, and overdamped motion. For underdamping, $x = A\, e^{(-bt/2m)} \cos(\omega d\, t + \phi)$, where $\omega d^2 = k/m - (b/2m)^2$.

$$v = dr/dt;\ a = dv/dt;\ v_{0x} = v_0 \cos(\theta);\ v_{0y} = v_0 \sin(\theta);\ x = x_0 + v_{0x}\, t;\ y = y_0 + v_{0y}\, t - (1/2)gt^2.$$

EXAMPLE If m = 1 kg, b = 2 N·s/m, k = 25 N/m, the roots are $-1 \pm i\sqrt{24}\ s^{-1}$, so the envelope decays as e^{-t}.

SENSEI NOTE The natural frequency $\sqrt{(k/m)}$ and damped oscillation frequency are not identical when $b > 0$.

KEY CONCEPT 2

Forced Steady State, Amplitude, and Phase

For $mx'' + bx' + kx = F_0 \cos(\omega t)$, a sinusoidal particular solution gives $A(\omega) = F_0/\sqrt{[(k-m\omega^2)^2+(b\omega)^2]}$ and $\tan\phi = b\omega/(k-m\omega^2)$, with the correct quadrant chosen. The homogeneous transient decays, leaving the forced response.

$$a_{rad} = v^2/r;\ a_{tan} = dv/dt;\ |a| = \sqrt{(a_{rad}^2 + a_{tan}^2)};\ v_{A/C} = v_{A/B} + v_{B/C}.$$

EXAMPLE At low ω the response is nearly in phase with the force; near resonance the phase changes rapidly; at high ω the displacement approaches opposition to the force.

SENSEI NOTE Phase is part of the physics of energy transfer; amplitude alone does not describe the response.

Ready to apply these ideas?

You've reinforced the essential concepts. Now strengthen your problem-solving skills by working through a few guided examples.

Continue to Guided Practice — page 7 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now apply the key concepts you've just reviewed. Work through these focused practice activities to reinforce your problem-solving skills before the final confidence check.

PRACTICE 1

Guided Example

Solve characteristic roots.

Solve $x'' + 4x' + 13x = 0$ and classify the motion.

PRACTICE 2

Independent Check

Evaluate forced amplitude and phase.

For $m=1$, $b=2$, $k=25$, $F_0=10$ and $\omega=5$ rad/s, find A and the phase lag.

Ready for a confidence check?

You've reinforced the essential skills through focused practice. Now confirm your understanding with two short questions.

Continue to Confidence Check — page 8 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've reinforced the key concepts and practiced the essential skills. Now confirm you're ready to move forward by completing these short confidence checks.

CONFIDENCE CHECK 1

Long-Time Solution

Identify what survives.

Why does the forced particular solution dominate at long times when $b > 0$?

CONFIDENCE CHECK 2

Resonance Condition

Distinguish ω_0 from displacement resonance.

For nonzero damping, is the maximum displacement amplitude generally exactly at ω_0?

HOW DID IT GO?

I answered ____ of 2 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've confirmed your understanding.

Take one final look at the essential ideas before choosing your next step.

Continue to Summary — page 10 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the essential ideas you'll want to remember.

KEY TAKEAWAY 1

Roots Encode the Free Dynamics

The characteristic roots determine whether the transient oscillates and how rapidly it decays.

KEY TAKEAWAY 2

A Particular Solution Encodes the Forced Response

The forcing frequency survives in steady state; amplitude and phase follow from the frequency-response denominator.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 14 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to check both the setup and the physical interpretation.

WARM-UP • ACTIVITY 1

Key Ideas

$mr^2 + br + k = 0$.

WHY IT WORKS Substitution of the exponential trial function gives the characteristic equation.

WARM-UP • ACTIVITY 2

Common Mistakes

The homogeneous/transient part.

WHY IT WORKS Initial conditions determine the homogeneous constants; forcing determines the particular response.

WARM-UP • ACTIVITY 3

Quick Application

Oscillation with an exponentially decaying envelope.

WHY IT WORKS The imaginary part gives oscillation and the negative real part gives decay.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Guided Example

$r = -2 \pm 3i$; $x = e^{(-2t)}[C_1 \cos(3t) + C_2 \sin(3t)]$; underdamped.

WHY IT WORKS The discriminant is 16 - 52 = -36, so the roots are complex with negative real part.

PRACTICE 2

Independent Check

$A = 10/10 = 1.00$ m; $k - m\omega^2 = 0$, so $\phi = \pi/2$.

WHY IT WORKS At $\omega = \sqrt{k/m}$, the reactive spring and inertia terms cancel; damping sets the finite amplitude and the displacement lags the force by 90°.

SOLUTIONS • CONFIDENCE CHECK

CONFIDENCE CHECK 1

Long-Time Solution

Because all stable homogeneous modes decay exponentially while the external force continuously sustains the particular solution.

WHY IT WORKS Positive damping gives negative real parts for stable homogeneous roots.

CONFIDENCE CHECK 2

Resonance Condition

No. It occurs slightly below ω_0 when a displacement-amplitude maximum exists.

WHY IT WORKS Differentiating $A(\omega)$ shows damping shifts the displacement-resonance frequency below the undamped natural frequency.

READINESS GUIDE

2 correct: You're ready to continue. • 1 correct: Review the missed idea, then continue. • 0 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this focused review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the focused review again to strengthen the essential concepts.	Build confidence with additional practice problems.	Continue to the next topic or learning resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com