FULL REVIEW

Full Review: Damping, Driving, and Resonance — Algebra-Based

Review damping, driven oscillations, resonance, phase, and energy transfer using the Algebra-Based treatment.

◷ **TIME** 45–60 minutes	◎ **BEST FOR** A complete topic review	✓ **FINISH WITH** A readiness check

After this full review, you'll be able to…
calculate damping and response quantities, compare driving and natural frequencies, and interpret resonance quantitatively.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • COURSE ALIGNMENT

Course Alignment

This Physics Sensei Unit Review supports MEC-U10 — Damping, Driving, and Resonance. Use it to reinforce concepts, prepare for homework, or review before a quiz or exam.

UNIT: MEC-U10 **TOPIC: Damping, Driving, and Resonance** **TREATMENT: Algebra-Based** **RESOURCE: Physics Sensei Unit Review**	**BEST USED** ✓ After reading the chapter ✓ Before starting homework ✓ Before a quiz or exam

Explore Physics Sensei →

Physics Sensei is an independent physics learning resource. This review uses original Physics Sensei organization, explanations, examples, and practice.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Compute the natural scale.

For $m = 0.80$ kg and $k = 72$ N/m, find ω_0.

ACTIVITY 2

Recall Activity 2

Classify with b_c.

For the same oscillator, find b_c and classify $b = 8.0$ N·s/m.

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Estimate resonance sharpness.

If $m = 1.0$ kg, $\omega_0 = 12$ rad/s, and $b = 3.0$ N·s/m, estimate Q.

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Free Damped Oscillations

For viscous damping Fd = -bv, the free equation is mx″ + bx′ + kx = 0. The ideal natural frequency is $\omega_0 = \sqrt{(k/m)}$, critical damping is b_c = 2√(km), and the underdamped amplitude envelope behaves as $A = A_0 e^{(-\gamma t)}$ with γ = b/(2m). Mechanical energy scales as A^2, so $E = E_0 e^{(-2\gamma t)}$.

$$r(t) = \langle x(t), y(t), z(t)\rangle;\ \text{displacement} = r_2 - r_1;\ v_{avg} = \text{displacement/elapsed time};\ v(t) = dr/dt;\ a(t) = dv/dt;\ \text{speed} = \sqrt{(v_x^2 + v_y^2 + v_z^2)}.$$

EXAMPLE For m=2.0 kg, k=50 N/m, b=4.0 N·s/m: ω_0=5.0 rad/s, b_c=20 N·s/m, γ=1.0 s^{-1}. After 1 s the amplitude is 0.368 A_0 but the energy is 0.135 E_0.

SENSEI NOTE Amplitude and energy do not have the same decay exponent.

KEY CONCEPT 2

Driven Amplitude and Resonance

For $F=F_0\cos(\omega t)$, the steady displacement amplitude is $A = F_0/\sqrt{[(k-m\omega^2)^2+(b\omega)^2]}$. Near resonance, the denominator becomes small, but damping keeps the amplitude finite. Increasing b lowers and broadens the peak.

$$v_{0x} = v_0 \cos(\theta);\ v_{0y} = v_0 \sin(\theta);\ x = x_0 + v_{0x} t;\ y = y_0 + v_{0y} t - (1/2)gt^2;\ v_y = v_{0y} - gt.\ \text{For equal heights: } T = 2v_{0y/g},\ R = v_0^2 \sin(2\theta)/g,\ H = v_{0y}^2/(2g).$$

EXAMPLE For m=1 kg, k=100 N/m, b=4 N·s/m, F_0=10 N, at ω=10 rad/s the amplitude is 0.250 m.

SENSEI NOTE At $\omega=\omega_0$, the spring and inertia terms cancel, but the displacement-resonance maximum of a damped system is generally slightly below ω_0.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Quality Factor, Bandwidth, and Phase

For weak damping, $Q \approx m\omega_0/b$ and $\Delta\omega \approx \omega_0/Q \approx b/m$. Larger Q means weaker relative damping and narrower bandwidth. The phase lag also changes across the response: nearly in phase at low frequency, around a quarter cycle near the natural scale, and approaching opposition at high frequency.

$$a_{rad} = v^2/r = \omega^2 r \text{ (inward)}; \; a_{tan} = dv/dt; \; |a| = \sqrt{(a_{rad}^2 + a_{tan}^2)}; \; v_{A/C} = v_{A/B} + v_{B/C}.$$

EXAMPLE If ω_0=20 rad/s and Q=10, then $\Delta\omega \approx 2$ rad/s.

SENSEI NOTE A tall resonance peak and a narrow bandwidth are two views of the same weak-damping behavior.

Ready to apply these ideas?

You’ve reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Find regime and decay.

A 1.5 kg oscillator has k=96 N/m and b=6.0 N·s/m. Find ω_0, b_c, γ, and A/A_0 after 0.75 s.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Evaluate frequency response.

For $m=1.0$ kg, $k=64$ N/m, $b=2.0$ N·s/m, $F_0=8.0$ N, find A at $\omega=8.0$ rad/s.

PRACTICE 3

Independent Problem

Connect Q and bandwidth.

An oscillator has $m=0.50$ kg, $\omega_0=30$ rad/s, and $b=1.5$ N·s/m. Estimate Q and $\Delta\omega$.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Classify a Damped Oscillator

Compute before naming the regime.

For $m=2.0$ kg, $k=18$ N/m, $b=15$ N·s/m, classify the damping.

QUICK CHECK 2

Amplitude vs Energy Decay

Use the squared-amplitude relation.

If an underdamped amplitude falls to 30% of its initial value, what fraction of mechanical energy remains?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Driven Response

Predict the effect of damping.

If b is doubled while m, k, F_0 and driving frequency near resonance stay fixed, what happens qualitatively to the response peak?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Use ω_0, b_c, and γ to Describe Free Decay

These quantities set the natural timescale, damping regime, and exponential envelope.

KEY TAKEAWAY 2

Use the Frequency-Response Denominator for Driven Amplitude

Resonance reflects competition among spring, inertia, and damping terms.

KEY TAKEAWAY 3

Use Q and Bandwidth to Measure Selectivity

Large Q means weak damping, slow decay, and a narrow resonance response.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to identify the step that supports the result.

WARM-UP • ACTIVITY 1

Recall Activity 1

$\omega_0 = \sqrt{(72/0.80)} = \sqrt{90} = 9.49$ rad/s.

WHY IT WORKS The ideal spring-mass natural angular frequency is $\sqrt{(k/m)}$.

WARM-UP • ACTIVITY 2

Recall Activity 2

$b_c = 2\sqrt{(0.80 \times 72)} = 15.2$ N·s/m; $b < b_c$, so it is underdamped.

WHY IT WORKS The damping coefficient must be compared with the critical value for that m and k.

WARM-UP • ACTIVITY 3

Recall Activity 3

$Q \approx m\omega_0/b = 4.0$.

WHY IT WORKS Q measures weak-damping frequency selectivity; larger Q means a sharper resonance.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

ω_0=8.00 rad/s; b_c=24.0 N·s/m; γ=2.00 s^{-1}; A/A_0=e^-1.5=0.223.

WHY IT WORKS Use $\omega_0=\sqrt{k/m}$, $b_c=2\sqrt{km}$, $\gamma=b/(2m)$, then the exponential envelope.

PRACTICE 2

Guided Problem

$A=8/\sqrt{[0^2+(16)^2]}=0.500$ m.

WHY IT WORKS Here $\omega=\omega_0$, so $k-m\omega^2=0$ and damping alone sets the denominator.

PRACTICE 3

Independent Problem

$Q\approx(0.50\times30)/1.5=10$; $\Delta\omega\approx30/10=3.0$ rad/s.

WHY IT WORKS Weak-damping relations connect time-domain damping to frequency-domain resonance width.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Classify a Damped Oscillator

$b_c=2\sqrt{36}=12$ N·s/m; $b>b_c$, so it is overdamped.

WHY IT WORKS The regime depends on the relative size of b and b_c.

QUICK CHECK 2

Amplitude vs Energy Decay

$0.30^2=0.090$, or 9%.

WHY IT WORKS For harmonic motion energy is proportional to amplitude squared.

QUICK CHECK 3

Driven Response

It becomes lower and broader, and Q decreases.

WHY IT WORKS More damping increases dissipation and reduces resonance selectivity.

READINESS GUIDE

3 correct: You’re ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the key ideas and examples again.	Continue with additional practice for this topic.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com