FULL REVIEW

Full Review: Damping, Driving, and Resonance — Calculus-Based

Review damping, driven oscillations, resonance, phase, and energy transfer using the Calculus-Based treatment.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
45–60 minutes	A complete topic review	A readiness check

After this full review, you'll be able to…
analyze damped and driven oscillator equations, connect transient and steady-state solutions, and interpret amplitude and phase response.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • COURSE ALIGNMENT

Course Alignment

This Physics Sensei Unit Review supports MEC-U10 — Damping, Driving, and Resonance. Use it to reinforce concepts, prepare for homework, or review before a quiz or exam.

UNIT: MEC-U10 **TOPIC: Damping, Driving, and Resonance** **TREATMENT: Calculus-Based** **RESOURCE: Physics Sensei Unit Review**	**BEST USED** ✓ After reading the chapter ✓ Before starting homework ✓ Before a quiz or exam

Explore Physics Sensei →

Physics Sensei is an independent physics learning resource. This review uses original Physics Sensei organization, explanations, examples, and practice.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Build the characteristic equation.

For $mx''+bx'+kx=0$ with $x=e^{(rt)}$, what polynomial equation must r satisfy?

ACTIVITY 2

Recall Activity 2

Interpret root type.

What does $b^2-4mk<0$ imply?

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Identify the forced frequency.

For a sinusoidal driver $F_0\cos(\omega t)$, what angular frequency appears in the long-time particular solution?

Ready to strengthen your understanding?

You’ve refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Characteristic Roots and Free Damping

The free equation $mx''+bx'+kx=0$ has characteristic roots $r=[-b\pm\sqrt{(b^2-4mk)}]/(2m)$. Negative real roots give decay. Complex roots $r=-\gamma\pm i\omega d$ produce $x=e^{(-\gamma t)}[C_1\cos(\omega d\ t)+C_2\sin(\omega d\ t)]$, with $\gamma=b/(2m)$ and $\omega d^2=k/m-\gamma^2$. Repeated roots give critical damping.

$$\mathbf{r(t) = \langle x(t), y(t), z(t)\rangle;\ displacement = r_2 - r_1;\ v_{avg} = displacement/elapsed\ time;\ v(t) = dr/dt;\ a(t) = dv/dt;\ speed = \sqrt{(v_x^2 + v_y^2 + v_z^2)}.}$$

EXAMPLE For $x''+4x'+13x=0$, $r=-2\pm3i$ and $x=e^{-2t}[C_1\cos3t+C_2\sin3t]$.

SENSEI NOTE The imaginary part sets oscillation; the real part sets the envelope decay.

KEY CONCEPT 2

Forced Particular Solution, Amplitude, and Phase

For $mx''+bx'+kx=F_0\cos(\omega t)$, choose a sinusoidal particular solution or use complex exponentials. The steady-state amplitude is $A=F_0/\sqrt{[(k-m\omega^2)^2+(b\omega)^2]}$. The phase lag satisfies $\tan\phi=b\omega/(k-m\omega^2)$, interpreted with the correct quadrant.

$$\mathbf{v_{0x} = v_0 \cos(\theta);\ v_{0y} = v_0 \sin(\theta);\ x = x_0 + v_{0x} t;\ y = y_0 + v_{0y} t - (1/2)gt^2;\ v_y = v_{0y} - gt.\ For\ equal\ heights:\ T = 2v_{0y/g},\ R = v_0^2 \sin(2\theta)/g,\ H = v_{0y}^2/(2g).}$$

EXAMPLE For $m=1$, $b=2$, $k=25$, $F_0=10$, $\omega=5$: $A=1.00$ m and $\phi=\pi/2$.

SENSEI NOTE At the undamped natural frequency, damping prevents divergence and sets a finite response.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Resonance, Power, and Frequency Selectivity

The displacement-amplitude maximum of a damped oscillator occurs at $\omega r=\sqrt{(\omega_0^2-2\gamma^2)}$ when that expression is meaningful. Average power input equals average dissipation in steady state. For weak damping, $Q\approx\omega_0/(2\gamma)=m\omega_0/b$ and bandwidth $\Delta\omega\approx\omega_0/Q$.

$$a_{rad} = v^2/r = \omega^2 r \text{ (inward)}; a_{tan} = dv/dt; |a| = \sqrt{(a_{rad}^2 + a_{tan}^2)}; v_{A/C} = v_{A/B} + v_{B/C}.$$

EXAMPLE As γ increases, ωr shifts downward, the peak decreases, and the bandwidth grows.

SENSEI NOTE Different resonance definitions (displacement amplitude, velocity, power) need not peak at exactly the same frequency when damping is finite.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Solve an underdamped IVP.

Solve $x''+2x'+10x=0$ with $x(0)=0.10$ m and $x'(0)=0$.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Evaluate amplitude and phase.

For $m=2$ kg, $b=4$ N·s/m, $k=50$ N/m, $F_0=12$ N, and $\omega=4$ rad/s, find A and ϕ.

PRACTICE 3

Independent Problem

Find displacement resonance.

For $m=1$ kg, $k=100$ N/m, $b=4$ N·s/m, find γ, ω_0, and ωr.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Read the Homogeneous Roots

Classify from roots directly.

If $r=-3\pm4i\ s^{-1}$, what is the qualitative motion?

QUICK CHECK 2

Separate Natural and Driving Frequencies

Identify the long-time frequency.

A system has $\omega_0=8$ rad/s but is driven at $\omega=11$ rad/s. What frequency remains after transients decay?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Check the Resonance Shift

Use the damped resonance formula.

If ω_0=10 rad/s and γ=1 rad/s, is ωr above or below ω_0?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Characteristic Roots Encode the Transient

Root real parts determine decay; imaginary parts determine damped oscillation.

KEY TAKEAWAY 2

Complex Dynamic Stiffness Encodes the Forced Response

The terms $k-m\omega^2$ and $b\omega$ determine amplitude and phase at every driving frequency.

KEY TAKEAWAY 3

Finite Damping Shifts and Broadens Resonance

The displacement peak moves slightly below ω_0 and becomes less sharp as damping increases.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to identify the step that supports the result.

WARM-UP • ACTIVITY 1

Recall Activity 1

$mr^2+br+k=0$.

WHY IT WORKS Substituting the exponential trial form converts the differential equation into an algebraic equation for the modes.

WARM-UP • ACTIVITY 2

Recall Activity 2

Complex conjugate roots: underdamped oscillation with an exponentially decaying envelope.

WHY IT WORKS The discriminant controls whether the homogeneous modes oscillate.

WARM-UP • ACTIVITY 3

Recall Activity 3

ω.

WHY IT WORKS A linear time-invariant oscillator responds in steady state at the driving frequency.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

$x=e^{-t}[C_1\cos3t+C_2\sin3t]$; $C_1=0.10$, $C_2=0.0333$, so $x=0.10e^{-t}\cos3t+0.0333e^{-t}\sin3t$.

WHY IT WORKS Roots are $-1\pm3i$. Apply $x(0)=C_1$ and $x'(0)=-C_1+3C_2=0$.

PRACTICE 2

Guided Problem

$k-m\omega^2=18$; $b\omega=16$; denominator$=\sqrt{580}=24.08$; $A=0.498$ m; $\phi=\text{atan2}(16,18)=41.6°$.

WHY IT WORKS The complex dynamic stiffness has real part $k-m\omega^2$ and imaginary part $b\omega$.

PRACTICE 3

Independent Problem

$\gamma=2\ \text{s}^{-1}$; $\omega_0=10$ rad/s; $\omega r=\sqrt{(100-8)}=9.59$ rad/s.

WHY IT WORKS Use $\gamma=b/(2m)$ and $\omega r=\sqrt{(\omega_0^2-2\gamma^2)}$.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Read the Homogeneous Roots

Underdamped oscillation at 4 rad/s with an envelope proportional to e^-3t.

WHY IT WORKS Complex roots give oscillation; their negative real part gives exponential decay.

QUICK CHECK 2

Separate Natural and Driving Frequencies

11 rad/s.

WHY IT WORKS The particular solution is locked to the forcing frequency.

QUICK CHECK 3

Check the Resonance Shift

$\omega r=\sqrt{98}\approx 9.90$ rad/s, below ω_0.

WHY IT WORKS Finite damping shifts the displacement-amplitude maximum downward.

READINESS GUIDE

3 correct: You’re ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the key ideas and examples again.	Continue with additional practice for this topic.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	Go to Practice →	Continue →

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com