FULL REVIEW

Full Review: Gravitation — Calculus-Based

Review the essential ideas, relationships, and problem-solving tools for Gravitation.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
45–60 minutes	A complete topic review	A readiness check

After this full review, you'll be able to…
recall the essential ideas, apply them to representative problems, and determine what to study next.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • TOPIC ALIGNMENT

Topic Alignment

This bundle is aligned to the approved Physics Sensei topic specification below. Use it to recover the topic structure, reinforce key decisions, and confirm readiness for the next study task.

TEXTBOOK: Independent Physics Sensei Unit Review
CHAPTER: Mechanics • MEC-U13
TOPIC: Gravitation
COURSE LEVEL: Calculus-Based

BEST USED

- ✓ After reading the chapter
- ✓ Before starting homework
- ✓ Before a quiz or exam

Review Prerequisites →

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Use scaling or limiting behavior.

For $g(r) = GM/r^2$, state dg/dr and explain what its sign says outside the source.

ACTIVITY 2

Recall Activity 2

Connect potential and field.

If $V(r) = -GM/r$, evaluate $-dV/dr$ and reconcile it with the radial vector field.

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Distinguish a special orbit from a general one.

In an ellipse, which distance enters Kepler's third-law form $T^2 = (4\pi^2/GM)(\text{distance})^3$?

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let’s rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Force, field, direction, and superposition

Use $\vec{g} = -GM\,\vec{r}/r^3$ for a point source and integrate $d\vec{g} = -G\,\vec{r}_{rel}\,dm/|\vec{r}_{rel}|^3$ for a continuous distribution. Symmetry should remove components before integration.

$$\vec{g}(\vec{r}) = -G\int(\vec{r}-\vec{r}')/|\vec{r}-\vec{r}'|^3\,dm.$$

EXAMPLE A ring of mass M and radius a produces axial field $g_x = -GMx/(x^2+a^2)^{(3/2)}$, toward the ring plane for $x > 0$.

SENSEI NOTE Lock the model before calculating: identify the source, test object, center-to-center distance, and direction of every contribution.

KEY CONCEPT 2

Potential, potential energy, work, and binding

Potential follows from work: $U(r)-U(\infty) = -\int_\infty^r \vec{F}\cdot d\,\vec{r}$. The field satisfies $\vec{g} = -\nabla V$ and force satisfies $\vec{F} = -\nabla U$.

$$U(r) = -\int_\infty^r \vec{F}\cdot d\,\vec{r} = -GMm/r;\ \vec{g} = -\nabla V.$$

EXAMPLE For $V = -GM/r$, $dV/dr = +GM/r^2$ and $\vec{g} = -(dV/dr)e_r = -(GM/r^2)e_r$.

SENSEI NOTE Negative U is not “negative energy motion.” K remains nonnegative; the sign of total E classifies whether an ideal two-body orbit is bound.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Circular orbits, escape, and Kepler relationships

Derive circular results from the inverse-square field and use angular momentum conservation to interpret equal areas. For a dominant central mass, specific energy $\varepsilon = v^2/2 - GM/r = -GM/(2a)$ for a bound Kepler orbit.

$$h = |\vec{r}\times\vec{v}| = \text{constant};\ \varepsilon = v^2/2 - GM/r = -GM/(2a);\ dA/dt = h/2.$$

EXAMPLE Angular momentum conservation makes an object move faster near periapsis and slower near apoapsis while sweeping equal areas in equal times.

SENSEI NOTE Never use altitude in an orbital formula until you convert it to radius from the center. Never add a separate “centripetal force” to gravity.

Ready to apply these ideas?

You’ve reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Model the interaction, show direction, and calculate or justify the result.

Derive the axial field of a uniform ring of mass M and radius a at coordinate x.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Choose an energy reference and solve without using constant-g kinematics.

Starting from $F_r = -GMm/r^2$, derive $U(r)$ with $U(\infty) = 0$.

PRACTICE 3

Independent Problem

Select the orbital relationship, state assumptions, and check scale.

Show why Kepler's second law follows from a central force.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Force or field?

Answer and justify in one sentence.

What is the limiting behavior of the ring's axial field at $x = 0$ and $x \gg a$?

QUICK CHECK 2

Energy sign and motion

Classify the claim.

If specific mechanical energy is zero, what asymptotic escape condition follows?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Orbit model check

Choose the valid relationship.

For a bound Kepler orbit, how are total energy and semimajor axis related?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You’ve checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Choose the gravitational quantity

Force depends on source and test masses; field and potential describe the source at a location. Preserve vector direction for force and field.

KEY TAKEAWAY 2

Use the general energy model

With zero at infinity, $U = -GMm/r$. Use mgh only for small height changes in an approximately uniform field.

KEY TAKEAWAY 3

Orbit means gravity-driven free fall

For a circular orbit, gravity is the net inward force. Radius is measured from the center; escape and general bound orbits are best analyzed with energy.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You’ve reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Replace each placeholder with a concise answer and the essential explanation.

WARM-UP • ACTIVITY 1

Recall Activity 1

$dg/dr = -2GM/r^3$; field magnitude decreases with increasing r.

WHY IT WORKS The negative derivative describes a decreasing magnitude, not the inward vector direction by itself.

WARM-UP • ACTIVITY 2

Recall Activity 2

$-dV/dr = -GM/r^2$; therefore $\vec{g} = (-GM/r^2)\ e_r$.

WHY IT WORKS The radial component is negative when outward e_r is positive, so the field points inward.

WARM-UP • ACTIVITY 3

Recall Activity 3

The semimajor axis a, not the instantaneous radius.

WHY IT WORKS A circular orbit is the special case a = r.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

$\vec{g}$ = -(GMx/(x^2+a^2)^(3/2)) e_x for $x > 0$ with e_x outward from the ring plane.

WHY IT WORKS All transverse components cancel. Each dm contributes an axial fraction x/√(x^2+a^2), and all elements share the same distance √(x^2+a^2).

PRACTICE 2

Guided Problem

U(r)-0 = -∫∞^r(-GMm/r′²)dr′ = -GMm/r.

WHY IT WORKS The integration limit and sign convention produce a negative potential for finite r.

PRACTICE 3

Independent Problem

Central force gives zero torque, so angular momentum L is constant; dA/dt = L/(2m) is constant.

WHY IT WORKS The infinitesimal swept area is dA = (1/2)$|\vec{r}\times\vec{v}|$dt.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Force or field?

$g_x = 0$ at $x = 0$; for $x \gg a$, $g_x \approx -GM/x^2$.

WHY IT WORKS Symmetry cancels at the center; far away the ring behaves like a point mass.

QUICK CHECK 2

Energy sign and motion

The object approaches $r \to \infty$ with $v \to 0$ in the ideal minimum-escape case.

WHY IT WORKS At infinity both $-GM/r$ and the minimum final kinetic energy approach zero.

QUICK CHECK 3

Orbit model check

$E = -GMm/(2a)$.

WHY IT WORKS More negative total energy corresponds to a smaller semimajor axis.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the key ideas and examples again.	Continue with additional practice for this topic.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com