FULL REVIEW

Full Review: Motion in Two Dimensions

Review two-dimensional kinematics through derivatives, integrals, motion graphs, constant acceleration, and variable-acceleration models.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
60 minutes	A complete unit review	A readiness check

After this full review, you'll be able to…
move fluently among $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$, use component integrals, and analyze trajectories parametrically.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 60 minutes

FR-02 • UNIT ALIGNMENT

Unit Alignment

This bundle is aligned to the approved Physics Sensei unit specification below. Use it to recover the unit structure, reinforce key decisions, and confirm readiness for the next study task.

ARCHITECTURE: Physics Sensei Independent Mechanics UNIT: MEC-U03 — Motion in Two Dimensions SCOPE: Unit Review PHYSICS LEVEL: Calculus-Based	BEST USED ✓ Before calculus-based vector-motion homework ✓ Before a quiz or exam ✓ When vector derivatives, parametric motion, or component integrals need reinforcement

Review Prerequisites →

Physics Sensei is an independent educational resource. This review is independently authored and organized under the approved Physics Sensei mechanics architecture.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 60 minutes

1 **Warm-Up Check — page 4** Activate prior knowledge.	2 **Core Concepts — page 6** Review the essential ideas.	3 **Guided Practice — page 8** Apply what you learned.
4 **Confidence Check — page 10** Confirm your understanding.	5 **Summary — page 12** Review the key ideas.	6 **Next Step — page 16** Continue your learning.

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Differentiate a vector position function component by component.

If $\vec{r}(t)=\langle 2t, 5t-4.9t^2\rangle$, write $\vec{v}(t)$ and $\vec{a}(t)$.

ACTIVITY 2

Recall Activity 2

Use a vector integral to accumulate displacement.

If $\vec{v}(t)$ is known, what integral gives $\Delta\vec{r}$ from t_1 to t_2?

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Use velocity components to identify a horizontal tangent.

What condition on v_y identifies the top of a projectile trajectory?

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Velocity and acceleration are vector derivatives

For planar motion, differentiate each position component to obtain velocity and differentiate again for acceleration.

$$\Delta x = x_2 - x_1 \quad \bullet \quad v\_avg = \Delta x/\Delta t$$

EXAMPLE $\vec{v}=d\vec{r}/dt$ and $\vec{a}=d^2\vec{r}/dt^2$. If $\vec{r}=\langle 2t,5t-4.9t^2\rangle$, then $\vec{v}=\langle 2,5-9.8t\rangle$ and $\vec{a}=\langle 0,-9.8\rangle$.

SENSEI NOTE Velocity is tangent to the trajectory; zero v_y does not imply zero vector velocity.

KEY CONCEPT 2

Vector integrals reconstruct displacement and velocity change

Integrate each vector component over the same time interval to reconstruct displacement or velocity change.

$$v = v_0 + at \quad \bullet \quad \Delta x = v_0 t + \tfrac{1}{2}at^2$$

EXAMPLE $\Delta\vec{r}=\int\vec{v}dt$ and $\Delta\vec{v}=\int\vec{a}dt$. Component integration preserves vector direction information.

SENSEI NOTE $\int|\vec{v}|dt$ gives path length; $|\int\vec{v}dt|$ gives displacement magnitude. They are generally different.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Projectile motion is a vector-valued integrated model

For an ideal projectile, $\vec{a}=\langle 0,-g\rangle$. Integrating produces the component velocity and parametric trajectory equations.

slope(x–t) → v • slope(v–t) → a • area(v–t) → Δx

EXAMPLE $\vec{r}(t)=\vec{r}_0+\vec{v}_0 t+\frac{1}{2}\vec{a}t^2$ and $\vec{v}(t)=\vec{v}_0+\vec{a}t$ for constant vector acceleration.

SENSEI NOTE Eliminate the parameter t only after establishing a physically consistent component model.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Differentiate a vector position function and evaluate speed and direction.

A particle has $\vec{r}(t)=\langle 3t^2, 4t\rangle$ m. Find $\vec{v}(t)$, $\vec{a}(t)$, and its speed at $t=2$ s.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Integrate each acceleration component using vector initial conditions.

A particle has $\vec{a}(t)=\langle 2t,-4\rangle$, $\vec{v}(0)=\langle 1,3\rangle$, $\vec{r}(0)=\langle 0,2\rangle$. Find $\vec{v}(t)$ and $\vec{r}(t)$.

PRACTICE 3

Independent Problem

Integrate speed to find path length and compare with displacement.

For $0\le t\le 2$ s, $\vec{v}(t)=\langle 3,4t\rangle$ m/s. Find displacement and express the path length as an integral.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Differentiate the vector function

Find velocity from $\vec{r}(t)$.

For $\vec{r}(t)=\langle t^2,3t\rangle$, find $\vec{v}$ at $t=2$ s.

QUICK CHECK 2

Integrate vector acceleration

Apply both initial velocity components.

If $\vec{a}(t)=\langle 2t,-g\rangle$ and $\vec{v}(0)=\langle 1,5\rangle$, write $\vec{v}(t)$.

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Connect velocity to the trajectory

Use dy/dx=(dy/dt)/(dx/dt).

How do you find the slope of a parametric trajectory from velocity components?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You’ve checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Vector derivatives convert $\vec{r} \to \vec{v} \to \vec{a}$

Differentiate component functions while preserving the vector basis.

KEY TAKEAWAY 2

Vector integrals reconstruct motion

Component integrals accumulate vector displacement and velocity change.

KEY TAKEAWAY 3

Parametric trajectories encode two-dimensional motion

The shared parameter t connects the coordinate functions and the physical trajectory.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Compare your setup, signs, units, and interpretation with the explanations below.

WARM-UP • ACTIVITY 1

Recall Activity 1

$\vec{v}(t)=\langle 2,5-9.8t\rangle$; $\vec{a}(t)=\langle 0,-9.8\rangle$.

WHY IT WORKS Differentiate both vector components once and then twice.

WARM-UP • ACTIVITY 2

Recall Activity 2

$\Delta\vec{r}=\int[t_1,t_2]\vec{v}(t)dt$.

WHY IT WORKS Component integrals accumulate the vector change in position.

WARM-UP • ACTIVITY 3

Recall Activity 3

$v_y=0$; verify v_x and $\vec{a}$ separately.

WHY IT WORKS A horizontal trajectory tangent has $dy/dx=v_y/v_x=0$ when $v_y=0$ and $v_x\neq 0$.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

$\vec{v}=\langle 6t,4\rangle$; $\vec{a}=\langle 6,0\rangle$; speed at 2 s is $\sqrt{160}=4\sqrt{10}$ m/s.

WHY IT WORKS Differentiate each component and evaluate $|\vec{v}(2)|=\sqrt{(12^2+4^2)}$.

PRACTICE 2

Guided Problem

$\vec{v}=\langle 1+t^2,3-4t\rangle$; $\vec{r}=\langle t+t^3/3,2+3t-2t^2\rangle$.

WHY IT WORKS Integrate each component and apply the corresponding initial condition.

PRACTICE 3

Independent Problem

$\Delta\vec{r}=\langle 6,8\rangle$ m; path length = $\int_0^2\sqrt{(9+16t^2)}dt$.

WHY IT WORKS Integrate $\vec{v}$ componentwise for displacement; integrate $|\vec{v}|$ for path length.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Differentiate the vector function

$\langle 4,3\rangle$ m/s.

WHY IT WORKS $\vec{v}=\langle 2t,3\rangle$, so $\vec{v}(2)=\langle 4,3\rangle$ m/s.

QUICK CHECK 2

Integrate vector acceleration

$\langle 1+t^2,5-gt\rangle$ m/s.

WHY IT WORKS Integrate each acceleration component and add $\vec{v}(0)$.

QUICK CHECK 3

Connect velocity to the trajectory

$dy/dx=v_y/v_x$.

WHY IT WORKS The chain rule gives $dy/dx=(dy/dt)/(dx/dt)$.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

?	◎	✓
I'm Still Unsure	**I Need More Practice**	**I'm Ready**
Review the key ideas and examples again.	Continue with additional practice for this unit.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com