FULL REVIEW

Full Review: Newton's Laws and Free-Body Diagrams

Express Newton's laws as vector differential equations and use integration or differential-equation models when forces vary with time, position, or velocity.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
45–60 minutes	A complete unit review	A readiness check

After this full review, you'll be able to…
move fluently between free-body diagrams, vector differential equations, impulse, and variable-force motion models while preserving the physical meaning of every force.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • UNIT ALIGNMENT

Unit Alignment

This bundle is aligned to the approved Physics Sensei unit specification below. Use it to recover the unit structure, reinforce key decisions, and confirm readiness for the next study task.

ARCHITECTURE: Physics Sensei Independent Mechanics UNIT: MEC-U05 — Newton's Laws and Free-Body Diagrams SCOPE: Unit Review PHYSICS LEVEL: Calculus-Based	BEST USED ✓ Before calculus-based dynamics homework ✓ When forces vary with time or velocity ✓ When you need to connect FBDs to differential equations

Review Prerequisites →

Physics Sensei is an independent educational resource. This review is independently authored and organized under the approved Physics Sensei mechanics architecture.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

State Newton second law in differential form.

For constant mass, write Newton's second law using velocity and using momentum.

ACTIVITY 2

Recall Activity 2

Recognize what an ODE solution needs.

After a free-body diagram gives m dv/dt = F(t), what additional information is needed to determine a unique velocity function?

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Connect steady motion to zero net force.

For linear drag with downward positive, $m\,dv/dt = mg - bv$. What condition defines terminal speed?

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let’s rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

The free-body diagram becomes a vector differential equation

Choose a system and write ΣF_ext = dp/dt. For constant mass, each component gives m d^2r/dt^2 = ΣF(r,v,t). The FBD supplies the physics; calculus evolves the state.

ΣF_ext = dp/dt

EXAMPLE If F(t) = (6t i + 4 j) N on a 2 kg particle, then dv_x/dt = 3t and dv_y/dt = 2.

SENSEI NOTE Do not begin with integration before the force model and coordinate directions are correct.

KEY CONCEPT 2

Integrals connect force, momentum, velocity, and position

Integrating net force over time gives impulse and momentum change. Integrating acceleration gives velocity; integrating velocity gives position, with constants fixed by initial conditions.

m d^2r/dt^2 = ΣF(r,v,t)

EXAMPLE J = ∫ΣF dt = Δp. For constant mass, Δv = (1/m)∫ΣF dt.

SENSEI NOTE An integral is not decorative calculus; it is required when the force or acceleration varies over the interval.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Velocity- and position-dependent forces create differential-equation models

Drag, springs, and other variable forces require equations such as $m\,dv/dt = mg - bv$ or $m\ddot{x} = -kx$. Characteristic time or frequency emerges from the force law.

overturning moment = restoring moment

EXAMPLE Linear drag gives $v_t = mg/b$ and time constant $\tau = m/b$. A spring gives $\omega = \sqrt{k/m}$.

SENSEI NOTE State the sign convention and force direction before solving; most ODE sign errors begin in the FBD.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Integrate a time-dependent vector force twice.

A 2.0 kg particle starts from rest at the origin under $F(t) = (6t\ i + 4\ j)$ N. Find $v(2\ s)$ and $r(2\ s)$.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Solve a first-order drag equation and identify its time scale.

A 1.5 kg object falls from rest with linear drag $b = 3.0$ N·s/m. Take downward positive and use $g = 9.8\ m/s^2$. Find terminal speed and speed at $t = 1.0$ s.

PRACTICE 3

Independent Problem

Solve a position-dependent force model.

A 2.0 kg mass experiences $F_x = -8.0x$ N. If $x(0)=0.30$ m and $v(0)=0$, find $x(t)$ and the velocity at $t = \pi/4$ s.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You’ve rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Momentum form

Identify the general law.

What is the most general compact form of Newton's second law for a particle system with momentum p?

QUICK CHECK 2

Impulse from variable force

Integrate force over time.

A 2.0 kg particle initially at rest experiences F_x = 4t N from t=0 to 3.0 s. What is its final x-velocity?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Terminal condition

Read the steady state from the differential equation.

For $m\,dv/dt = mg - bv$ with downward positive, what equality holds at terminal speed?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You’ve checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

FBD first, differential equation second

Calculus evolves a correct physical model; it cannot repair a missing or misdirected force.

KEY TAKEAWAY 2

Integrals encode accumulated force effects

Impulse is the time integral of net force and determines momentum change.

KEY TAKEAWAY 3

Variable force laws set natural scales

Drag introduces $\tau = m/b$ and springs introduce $\omega = \sqrt{(k/m)}$, both derived directly from Newton second law.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Replace each placeholder with a concise answer and the essential explanation.

WARM-UP • ACTIVITY 1

Recall Activity 1

$\Sigma F = m\, dv/dt$ and, more generally, $\Sigma F = dp/dt$.

WHY IT WORKS Force determines the time rate of change of momentum. For constant mass, $dp/dt = m\, dv/dt$.

WARM-UP • ACTIVITY 2

Recall Activity 2

An initial velocity (or another equivalent velocity condition).

WHY IT WORKS Integrating acceleration introduces a constant of integration that is fixed by the initial condition.

WARM-UP • ACTIVITY 3

Recall Activity 3

$dv/dt = 0$, so $mg - bv_t = 0$.

WHY IT WORKS Terminal speed is the steady state of the differential equation: acceleration vanishes even though gravity and drag remain nonzero and balanced.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

$v(2) = (6\,i + 4\,j)$ m/s; $r(2) = (4\,i + 4\,j)$ m.

WHY IT WORKS $a = F/m = (3t\,i + 2\,j)$ m/s². Integrating with $v(0)=0$ gives $v = (1.5t^2\,i + 2t\,j)$. Integrating again with $r(0)=0$ gives $r = (0.5t^3\,i + t^2\,j)$. At t=2 s these are (6,4) m/s and (4,4) m.

PRACTICE 2

Guided Problem

$v_t = 4.90$ m/s; $v(1.0\text{ s}) = 4.24$ m/s downward.

WHY IT WORKS $m\,dv/dt = mg - bv$ gives $v(t)=v_t[1-e^{-t/\tau}]$, where $v_t=mg/b=4.90$ m/s and $\tau=m/b=0.50$ s. Thus $v(1)=4.90(1-e^{-2})=4.24$ m/s.

PRACTICE 3

Independent Problem

$x(t)=0.30\cos(2t)$ m; $v(\pi/4) = -0.60$ m/s.

WHY IT WORKS The equation is $2\ddot{x}=-8x$, so $\ddot{x}+4x=0$ and $\omega=2$ rad/s. The initial conditions give $x=0.30\cos(2t)$, $v=-0.60\sin(2t)$. At $t=\pi/4$, $\sin(\pi/2)=1$, so $v=-0.60$ m/s.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Momentum form

$\Sigma F\_ext = dp/dt$.

WHY IT WORKS The momentum form remains the fundamental statement; m dv/dt follows for constant mass.

QUICK CHECK 2

Impulse from variable force

9.0 m/s.

WHY IT WORKS Impulse $J = \int_0^3 4t\,dt = 18$ N·s = Δp. With m=2.0 kg and initial velocity zero, v = 18/2 = 9.0 m/s.

QUICK CHECK 3

Terminal condition

$bv\_t = mg$.

WHY IT WORKS At terminal speed dv/dt=0, so the drag force exactly balances weight while the object continues at constant velocity.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

?	◎	✓
I'm Still Unsure	**I Need More Practice**	**I'm Ready**
Review the key ideas and examples again.	Continue with additional practice for this unit.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com