FULL REVIEW

Full Review: Simple Harmonic Motion

Review the essential ideas, relationships, and problem-solving tools for simple harmonic motion.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
45–60 minutes	A complete topic review	A readiness check

After this full review, you'll be able to…

analyze SHM with differential equations and derivatives, use energy methods, and justify the small-angle pendulum approximation.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • COURSE ALIGNMENT

Course Alignment

This bundle is designed to complement the chapter listed below. Use it to reinforce key concepts, prepare for homework, or review before a quiz or exam.

TEXTBOOK: Independent Physics Sensei Unit Review **CHAPTER: Mechanics • Unit MEC-U09** **TOPIC: MEC-U09 — Simple Harmonic Motion** **COURSE LEVEL: Calculus-Based**	**BEST USED** ✓ After reading the chapter ✓ Before starting homework ✓ Before a quiz or exam

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FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Connect the force law to the equation of motion.

Starting from $F = -kx$ and Newton's second law, write the differential equation for a mass m attached to an ideal spring.

ACTIVITY 2

Recall Activity 2

Differentiate the position function carefully.

If $x(t) = A\cos(\omega t + \phi)$, write $v(t)$ and $a(t)$.

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Use the potential-energy picture near stable equilibrium.

For a conservative system with a stable equilibrium at $x = 0$, what feature of $U(x)$ makes small oscillations approximately simple harmonic?

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Restoring force and the SHM model

SHM is defined by $d^2x/dt^2 = -\omega^2 x$. For a spring-mass system, $\omega^2 = k/m$. More generally, linearization about a stable equilibrium gives the same form when the leading restoring term is proportional to displacement.

$$d^2x/dt^2 + \omega^2 x = 0;\ \text{spring: } \omega^2 = k/m;\ \text{near a stable minimum, } k_e = d^2U/dx^2 \text{ evaluated at equilibrium}$$

EXAMPLE If $U(x) \approx U_0 + \tfrac{1}{2}(18\ \text{N/m})x^2$ near equilibrium, then small motion has $\omega = \sqrt{18/m}$.

SENSEI NOTE Do not call every back-and-forth motion SHM. The key test is whether the restoring force is approximately linear in displacement over the motion being modeled.

KEY CONCEPT 2

Amplitude, period, phase, displacement, velocity, and acceleration

Differentiating $x(t)$ gives $v(t)$ and $a(t)$. Displacement and acceleration differ by π radians; velocity differs from displacement by $\pi/2$ radians. Phase-space and x-v-a graphs encode these relations without treating the three quantities as simultaneous maxima.

$$v = dx/dt = -A\omega \sin(\omega t + \phi);\ a = d^2x/dt^2 = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 x$$

EXAMPLE For $x = 0.040 \cos(6t + \pi/3)$ m, $v = -0.240 \sin(6t + \pi/3)$ m/s and $a = -1.44 \cos(6t + \pi/3)$ m/s^2.

SENSEI NOTE Do not confuse amplitude with distance traveled in a cycle. One full cycle covers a path length 4A, while the amplitude is only A.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Energy, spring oscillators, and the simple pendulum

Energy conservation follows directly from the SHM equation. For a pendulum, $\sin\theta \approx \theta$ converts $\ddot{\theta} = -(g/L)\sin\theta$ into $\ddot{\theta} + (g/L)\theta = 0$. This approximation is the mathematical reason the small-angle pendulum is simple harmonic.

$$\frac{d}{dt}\left[\tfrac{1}{2}m(dx/dt)^2 + \tfrac{1}{2}kx^2\right] = 0;\ \text{pendulum: } \ddot{\theta} + (g/L)\theta \approx 0 \text{ for } |\theta| \ll 1 \text{ rad}$$

EXAMPLE For $m = 0.50$ kg, $k = 200$ N/m, $A = 0.080$ m, at $x = 0.040$ m: $U = 0.16$ J, $K = 0.48$ J, and $|v| \approx 1.39$ m/s.

SENSEI NOTE The simple-pendulum formula is a small-angle result. At large amplitude, the motion is periodic but not exactly simple harmonic and the period increases with amplitude.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Identify equilibrium, choose the positive direction, and apply the restoring-force model.

A particle satisfies $d^2x/dt^2 = -36x$. Identify ω and write the general sinusoidal form for $x(t)$.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Use the cycle relationships among x, v, a, T, f, and ω.

For $x(t) = 0.030\cos(4t - \pi/6)$ m, find $v(0)$ and $a(0)$.

PRACTICE 3

Independent Problem

Use energy or the pendulum model, and state the assumptions that make the model valid.

Derive the small-angle pendulum angular frequency from $\ddot{\theta} = -(g/L)\sin\theta$.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Restoring-force model check

Answer and justify in one sentence.

What mathematical relation between acceleration and displacement defines SHM?

QUICK CHECK 2

Cycle and phase check

Use the cycle, not memorized slogans.

For $x = A\cos(\omega t)$, at what phase is v most negative?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Energy and pendulum model check

State the model assumption with the answer.

Why does the exact pendulum equation fail to be linear SHM at large angles?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You’ve checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Test the restoring law first

SHM requires a stable equilibrium and acceleration proportional to −displacement. For a spring, $F = -kx$ and $\omega = \sqrt{k/m}$.

KEY TAKEAWAY 2

Read the cycle through phase relationships

Use $\omega = 2\pi f$, $x = A\cos(\omega t + \phi)$, $v = dx/dt$, and $a = -\omega^2 x$. At equilibrium speed is maximum; at turning points speed is zero.

KEY TAKEAWAY 3

Use energy and respect the small-angle limit

For an ideal spring, $E = \frac{1}{2}kA^2$ is constant. A simple pendulum is approximately SHM only when $\sin\theta \approx \theta$ is valid.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to identify the step that supports the result.

WARM-UP • ACTIVITY 1

Recall Activity 1

$m\, d^2x/dt^2 = -kx$, or $d^2x/dt^2 + (k/m)x = 0$.

WHY IT WORKS The defining linear SHM equation has acceleration proportional to $-x$, with angular frequency squared equal to k/m.

WARM-UP • ACTIVITY 2

Recall Activity 2

$v = -A\omega \sin(\omega t + \phi)$; $a = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 x$.

WHY IT WORKS Velocity is one quarter-cycle out of phase with displacement; acceleration is exactly opposite displacement.

WARM-UP • ACTIVITY 3

Recall Activity 3

U has a local minimum and is approximately quadratic: $U \approx U_0 + \frac{1}{2}k_e x^2$, so $F \approx -k_e x$.

WHY IT WORKS A smooth potential near a stable minimum has a leading quadratic term, producing a linear restoring force.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

$\omega = 6$ rad/s; $x(t) = A\cos(6t + \phi)$, equivalently a sine-cosine combination.

WHY IT WORKS Compare the equation directly with $\ddot{x} = -\omega^2 x$.

PRACTICE 2

Guided Problem

$v(0) = -A\omega \sin(-\pi/6) = +0.060$ m/s; $a(0) = -A\omega^2 \cos(-\pi/6) \approx -0.416$ m/s^2.

WHY IT WORKS Differentiate the position function first, then substitute $t = 0$ and preserve the phase sign.

PRACTICE 3

Independent Problem

For small θ in radians, $\sin\theta \approx \theta$, so $\ddot{\theta} + (g/L)\theta = 0$ and $\omega = \sqrt{(g/L)}$.

WHY IT WORKS The pendulum is approximately SHM only after linearizing $\sin\theta$ about $\theta = 0$.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Restoring-force model check

$a = -\omega^2 x$, equivalently $\ddot{x} + \omega^2 x = 0$.

WHY IT WORKS The proportionality to $-x$ produces sinusoidal motion with constant angular frequency.

QUICK CHECK 2

Cycle and phase check

At $\omega t = \pi/2$ modulo 2π, because $v = -A\omega \sin(\omega t)$.

WHY IT WORKS Maximum negative velocity occurs when $\sin(\omega t) = 1$.

QUICK CHECK 3

Energy and pendulum model check

Because $\sin\theta$ is not proportional to θ at large angle, so the restoring torque is nonlinear.

WHY IT WORKS Only the small-angle approximation $\sin\theta \approx \theta$ gives a constant-coefficient linear SHM equation.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the key ideas and examples again.	Continue with additional practice for this topic.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	Go to Practice →	Continue →

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

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