FULL REVIEW

Full Review: Projectile Motion — Calculus-Based

Review the essential ideas, relationships, and problem-solving tools for projectile motion.

TIME	BEST FOR	FINISH WITH
45–60 minutes	A complete topic review	A readiness check

After this full review, you'll be able to…

analyze vector motion, solve representative projectile and relative-motion problems, and interpret circular acceleration.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • COURSE ALIGNMENT

Course Alignment

This bundle is designed to complement the chapter listed below. Use it to reinforce key concepts, prepare for homework, or review before a quiz or exam.

TEXTBOOK: Independent Physics Sensei Unit Review **CHAPTER: MEC-U15** **TOPIC: MEC-U15 — Projectile Motion** **COURSE LEVEL: Calculus-based introductory university physics**	**BEST USED** ✓ After reading the chapter ✓ Before starting homework ✓ Before a quiz or exam

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Physics Sensei is an independent educational resource for college physics review and practice.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

1 **Warm-Up Check — page 4** Activate prior knowledge.	2 **Core Concepts — page 6** Review the essential ideas.	3 **Guided Practice — page 8** Apply what you learned.
4 **Confidence Check — page 10** Confirm your understanding.	5 **Summary — page 12** Review the key ideas.	6 **Next Step — page 16** Continue your learning.

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Write the vector position function and differentiate component by component.

For ideal projectile motion with $r(0) = \langle 0, y_0 \rangle$, $v(0) = \langle v_0x, v_0y \rangle$, write $r(t)$, $v(t)$, and $a(t)$.

ACTIVITY 2

Recall Activity 2

Use a derivative condition to locate the apex.

For $y(t) = y_0 + v_0y\ t - ½gt^2$, what condition determines the time of maximum height?

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Eliminate the parameter t to obtain the path equation.

Given $x = v_0 \cos\theta \cdot t$ and $y = y_0 + v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$, what type of function y(x) results?

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Projectile Motion from Vector Differential Equations

The ideal projectile model follows from the vector equation $d^2r/dt^2 = \langle 0, -g \rangle$. Integrating with initial conditions produces the component velocity and position functions. This formulation makes the independence of x and y explicit and shows that the common parameter t connects the components.

$$\mathbf{a(t) = \langle 0, -g \rangle;\ v(t) = \langle v_0 \cos\theta,\ v_0 \sin\theta - gt \rangle;\ r(t) = \langle x_0 + v_0 \cos\theta \cdot t,\ y_0 + v_0 \sin\theta \cdot t - \tfrac{1}{2}gt^2 \rangle.}$$

EXAMPLE For $v_0 = 24.0$ m/s, $\theta = 40.0°$, and $y_0 = 1.50$ m, $r(t) = \langle 18.4t, 1.50 + 15.4t - 4.90t^2 \rangle$ m and $v(t) = \langle 18.4, 15.4 - 9.80t \rangle$ m/s.

SENSEI NOTE Differentiate vector functions component by component. Do not differentiate the speed magnitude and treat the result as the acceleration vector.

KEY CONCEPT 2

Trajectory Equation, Extrema, and Range

Eliminating time from the parametric equations gives the parabolic trajectory y(x). The apex follows from $dy/dt = 0$ or $dy/dx = 0$. Landing times are roots of the vertical coordinate equation. For equal-height launches, the nonzero root yields the familiar time and range formulas; unequal heights require the general root.

$$\mathbf{y(x) = y_0 + x \tan\theta - g\, x^2/(2v_0^2 \cos^2\theta);\ t\_top = v_0 \sin\theta/g;\ y\_max = y_0 + v_0^2 \sin^2\theta/(2g).}$$

EXAMPLE For a level launch at $v_0 = 28.0$ m/s and $\theta = 35.0°$, the nonzero root gives $T = 3.28$ s and $R = 75.2$ m; the apex is 13.2 m above launch.

SENSEI NOTE A parabolic y(x) trajectory is a consequence of constant downward acceleration and zero horizontal acceleration; it is not a universal property of motion through a fluid.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Velocity Geometry, Impact State, and Inverse Problems

The tangent to r(t) is the velocity vector. At any event time, speed is |v| and direction follows from the component ratio. In inverse problems, specified target coordinates impose simultaneous constraints on x(t) and y(t). Solving them can reveal required launch parameters or whether a target is reachable under the ideal model.

$$v = dr/dt;\ |v| = \sqrt{(v_x^2 + v_y^2)};\ \tan\phi = v_y/v_x;\ \text{target constraints: } x(t^*) = x\_T \text{ and } y(t^*) = y\_T.$$

EXAMPLE For the 24.0 m/s, 40.0°, y_0 = 1.50 m launch, the landing root is t = 3.24 s. Then v = <18.4, −16.4> m/s, |v| = 24.6 m/s, directed 41.7° below +x.

SENSEI NOTE Use atan2(v_y, v_x) or explicit quadrant reasoning for velocity direction; a simple arctangent can lose quadrant information.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Integrate the acceleration vector and apply initial conditions.

A projectile has $a(t) = \langle 0, -9.80\rangle\ m/s^2$, $r(0) = \langle 0, 2.00\rangle$ m, and $v(0) = \langle 14.0, 12.0\rangle$ m/s. Find $r(t)$ and $v(t)$.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Find the positive landing root, then evaluate the velocity vector there.

For $r(t) = \langle 18.4t, 1.50 + 15.4t - 4.90t^2\rangle$ m, find landing time, range, and impact velocity.

PRACTICE 3

Independent Problem

Eliminate time and identify the curvature of the trajectory.

A projectile is launched from the origin with speed v_0 at angle θ. Derive $y(x)$ for the ideal model.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Differentiate the Trajectory

Relate the slope of the path to the velocity components.

For a parametric projectile path, what is dy/dx in terms of the velocity components?

QUICK CHECK 2

Classify the Apex

Use first and second derivatives.

Why does $v_y = 0$ identify a maximum rather than a minimum for an ideal upward-launched projectile?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Parameter Elimination Check

Identify the source of the quadratic term.

Why does eliminating t from ideal projectile x(t) and y(t) produce a quadratic in x?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Start from a(t)

The vector equation a = <0, −g> integrates directly to the full projectile velocity and position functions.

KEY TAKEAWAY 2

The Path Is Parametric

x(t) and y(t) are linked by the same parameter; eliminating t produces the parabolic trajectory y(x).

KEY TAKEAWAY 3

Use Derivatives for Geometry

Velocity is tangent to the path, $v_y = 0$ locates the apex, and the second derivative confirms it is a maximum.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to identify the step that supports the result.

WARM-UP • ACTIVITY 1

Recall Activity 1

$r(t) = \langle v_0x\, t,\ y_0 + v_0y\, t - \tfrac{1}{2}gt^2\rangle$; $v(t) = \langle v_0x,\ v_0y - gt\rangle$; $a(t) = \langle 0, -g\rangle$.

WHY IT WORKS Integrating the constant acceleration vector $\langle 0, -g\rangle$ once gives velocity and a second time gives position, with the initial conditions fixing the constants.

WARM-UP • ACTIVITY 2

Recall Activity 2

$dy/dt = v_y(t) = 0$, so $t\_top = v_0y/g$.

WHY IT WORKS The vertical coordinate has an extremum when its first derivative vanishes. Since $d^2y/dt^2 = -g < 0$, the extremum is a maximum.

WARM-UP • ACTIVITY 3

Recall Activity 3

A downward-opening quadratic: $y = y_0 + x \tan\theta - [g x^2/(2v_0^2 \cos^2\theta)]$.

WHY IT WORKS Solve $t = x/(v_0 \cos\theta)$ from the horizontal equation and substitute it into $y(t)$. The x^2 term makes the ideal trajectory parabolic.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

$v(t) = \langle 14.0, 12.0 - 9.80t \rangle$ m/s; $r(t) = \langle 14.0t, 2.00 + 12.0t - 4.90t^2 \rangle$ m.

WHY IT WORKS Integrate acceleration once and use v(0), then integrate velocity and use r(0). The constants are fixed separately for x and y.

PRACTICE 2

Guided Problem

$t = 3.24$ s; range = 59.6 m; $v_{impact} = \langle 18.4, -16.4 \rangle$ m/s.

WHY IT WORKS Set $y(t) = 0$ and keep the positive root. Substitute that time into x(t) and v(t). The impact speed is 24.6 m/s.

PRACTICE 3

Independent Problem

$y(x) = x \tan\theta - g x^2/(2v_0^2 \cos^2\theta)$.

WHY IT WORKS From $x = v_0 \cos\theta \cdot t$, $t = x/(v_0 \cos\theta)$. Substitute this expression into $y = v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$ and simplify.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Differentiate the Trajectory

$dy/dx = (dy/dt)/(dx/dt) = v_y/v_x$, provided $v_x \neq 0$.

WHY IT WORKS The trajectory slope equals the ratio of vertical to horizontal velocity components because both coordinates share the same parameter t.

QUICK CHECK 2

Classify the Apex

Because $d^2y/dt^2 = -g < 0$ at all times.

WHY IT WORKS The first derivative vanishes at the apex and the negative second derivative makes y(t) concave downward, so the stationary point is a maximum.

QUICK CHECK 3

Parameter Elimination Check

Because x is linear in t while y contains a t^2 term; substituting $t \propto x$ converts t^2 into x^2.

WHY IT WORKS Zero horizontal acceleration makes x(t) linear. Constant vertical acceleration makes y(t) quadratic. Their combination yields a parabola.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the key ideas and examples again.	Continue with additional practice for this topic.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	Go to Practice →	Continue →

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

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