FOCUSED REVIEW

Focused Review: Center of Mass and Systems of Particles

Reinforce the highest-leverage center-of-mass models for discrete and continuous systems and connect them to system dynamics.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
15 minutes	Focused review	Greater confidence

After this focused review, you'll be able to…

choose a sum or integral for center of mass, use the correct differential mass element, and connect P = M VCM with dP/dt = net external force.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 15 minutes

FR-02 • PHYSICS SENSEI UNIT REVIEW

Physics Sensei Unit Review

Use this Physics Sensei Unit Review to reinforce key concepts, prepare for homework, and review before a quiz or exam.

UNIT: MEC-U19 **TOPIC FAMILY:** Mechanics **TOPIC:** Center of Mass and Systems of Particles **COURSE LEVEL:** Calculus-based introductory university physics	**BEST USED** ✓ **To reinforce key concepts** ✓ Before starting homework ✓ Before a quiz or exam

Continue with Physics Sensei →

Physics Sensei is an independent educational resource organized around core college-physics ideas, problem-solving models, and study workflows.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six focused stages in order. Each stage reinforces your understanding and prepares you for a final confidence check.

6 stages • Approximately 15 minutes

1 **Warm-Up Check — page 4** Refresh key ideas.	2 **Core Concepts — page 6** Reinforce the essentials.	3 **Guided Practice — page 7** Strengthen key skills.
4 **Confidence Check — page 9** Confirm your understanding.	5 **Summary — page 10** Remember the essentials.	6 **Next Step — page 14** Choose your next step.

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a minute to reactivate what you already know. This quick warm-up will help you focus on the most important ideas before moving on.

WARM-UP 1

Key Ideas

Recall the discrete and continuous center-of-mass expressions.

Write rCM for discrete particles and for a continuous distribution, and state the role of dm.

WARM-UP 2

Common Mistakes

Recall the momentum connection.

Starting from rCM for constant total mass, what relation follows after one time derivative?

FR-04 • WARM-UP CHECK CONTINUED

WARM-UP 3

Quick Application

Connect momentum rate to external force.

For constant M, complete the chain dP/dt = ____ = M ACM.

Ready to strengthen your understanding?

You've refreshed the most important ideas. Now reinforce the concepts you'll use most often.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's reinforce the most important concepts you'll need to solve problems with confidence. Focus on the ideas that appear most often in homework, quizzes, and exams.

KEY CONCEPT 1

Discrete and Continuous Center of Mass

Use a mass-weighted sum for discrete particles and replace the sum with an integral for continuous mass distributions. Build dm from the appropriate density, choose physical limits, and use symmetry before integrating when possible.

$$\mathbf{r}_{\mathrm{CM}} = \tfrac{1}{M}\sum_i m_i \mathbf{r}_i \qquad \mathbf{r}_{\mathrm{CM}} = \tfrac{1}{M}\int \mathbf{r}\,dm \qquad M = \int dm$$

EXAMPLE For a rod on $0 \le x \le L$ with $\lambda(x) = kx$, use $dm = kx\,dx$. Then $x_{CM} = [\int_0^L x(kx)dx]/[\int_0^L kx\,dx] = 2L/3$.

SENSEI NOTE Choose dm from the physical density model before integrating; the wrong mass element gives the wrong center of mass.

KEY CONCEPT 2

Center-of-Mass Dynamics

For constant total mass, differentiating rCM gives P = M VCM. Differentiating total momentum gives dP/dt = net external force, so net external force = M ACM. Internal forces do not appear in the net external-force equation for the complete system.

$$\frac{d\mathbf{P}}{dt} = \sum \mathbf{F}_{\mathrm{ext}} = M\mathbf{A}_{\mathrm{CM}}$$

EXAMPLE For constant M, differentiating rCM gives VCM = (1/M)Σmi vi, so P = M VCM. Differentiating P gives dP/dt = net external force = M ACM.

SENSEI NOTE Internal forces may redistribute momentum inside the system, but they do not appear in the net external-force equation for the complete system.

Ready to apply these ideas?

You've reinforced the essential concepts. Now strengthen your problem-solving skills by working through a few guided examples.

Continue to Guided Practice — page 7 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now apply the key concepts you've just reviewed. Work through these focused practice activities to reinforce your problem-solving skills before the final confidence check.

PRACTICE 1

Guided Example

Choose dm correctly and integrate the weighted position.

A thin rod occupies $0 \le x \le L$ with $\lambda(x) = kx$. Set up and evaluate x_{CM}.

PRACTICE 2

Independent Check

Differentiate the center-of-mass relation and interpret the result physically.

For a constant-mass particle system, derive $P = M V_{CM}$ and state what happens to V_{CM} when net external force = 0.

Ready for a confidence check?

You've reinforced the essential skills through focused practice. Now confirm your understanding with two short questions.

Continue to Confidence Check — page 9 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've reinforced the key concepts and practiced the essential skills. Now confirm you're ready to move forward by completing these short confidence checks.

CONFIDENCE CHECK 1

Continuous Center of Mass

Identify the correct density model, differential mass element, and integration limits.

For a nonuniform rod, what two ingredients must be correct before evaluating $x_{CM} = (1/M)\int x\, dm$*?*

CONFIDENCE CHECK 2

Momentum and External Force

Use dP/dt = net external force and the constant-mass relation $P = M V_{CM}$.

If net external force = 0 for a constant-mass system, what does dP/dt *imply about* P *and* V_{CM}*?*

HOW DID IT GO?

I answered ____ of 2 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've confirmed your understanding.

Take one final look at the essential ideas before choosing your next step.

Continue to Summary — page 10 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the essential ideas you'll want to remember.

KEY TAKEAWAY 1

Choose the Correct Mass Model

Use Σ for discrete particles and ∫ with the correct dm for continuous distributions.

KEY TAKEAWAY 2

Connect rCM, P, and External Force

For constant M, rCM → VCM → P and dP/dt = net external force → M ACM.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 14 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to check both the setup and the physical interpretation.

WARM-UP • ACTIVITY 1

Key Ideas

Discrete: $r_{CM} = (1/M)\Sigma m_i r_i$. Continuous: $r_{CM} = (1/M)\int r\, dm$.

WHY IT WORKS A discrete system is represented by separate masses m_i; a continuous body is represented by differential mass elements dm built from the relevant density.

WARM-UP • ACTIVITY 2

Common Mistakes

$P = M V_{CM}$.

WHY IT WORKS For constant M, differentiating $r_{CM} = (1/M)\Sigma m_i r_i$ gives $V_{CM} = (1/M)\Sigma m_i v_i$, and the numerator is total momentum P.

WARM-UP • ACTIVITY 3

Quick Application

net external force.

WHY IT WORKS Newton’s second law for the complete system gives dP/dt = net external force. With constant M, $dP/dt = M A_{CM}$.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Guided Example

xCM = 2L/3.

WHY IT WORKS dm = kx dx. Then $M = \int_0^L kx\,dx = kL^2/2$ and $\int_0^L x\,dm = \int_0^L kx^2\,dx = kL^3/3$, so xCM = 2L/3.

PRACTICE 2

Independent Check

P = M VCM, and if net external force = 0 then P and VCM remain constant.

WHY IT WORKS Differentiating the center-of-mass definition gives P = M VCM. Zero net external force gives dP/dt = 0, so P and therefore VCM are constant for constant M.

SOLUTIONS • CONFIDENCE CHECK

CONFIDENCE CHECK 1

Continuous Center of Mass

The differential mass element dm and the physical integration limits must both be correct.

WHY IT WORKS dm translates the density model into mass, and the limits define which portion of the body belongs to the chosen system.

CONFIDENCE CHECK 2

Momentum and External Force

dP/dt = 0, so P is constant. With constant M and P = M VCM, VCM is also constant.

WHY IT WORKS net external force = 0 implies dP/dt = 0. Constant total momentum together with constant mass means the center-of-mass velocity does not change.

READINESS GUIDE

2 correct: You're ready to continue. • 1 correct: Review the missed idea, then continue. • 0 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this focused review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the focused review again to strengthen the essential concepts.	Build confidence with additional practice problems.	Continue to the next topic or learning resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com