FULL REVIEW

Full Review: Center of Mass and Systems of Particles

Review center-of-mass models for discrete and continuous systems, system momentum, and the external-force dynamics of the center of mass.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
60 minutes	A complete unit review	A readiness check

After this full review, you'll be able to…
calculate center of mass for discrete and continuous distributions, connect total momentum to center-of-mass velocity, and derive how net external force governs center-of-mass acceleration.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 60 minutes

FR-02 • PHYSICS SENSEI UNIT REVIEW

Physics Sensei Unit Review

Use this Physics Sensei Unit Review to reinforce key concepts, prepare for homework, and review before a quiz or exam.

UNIT: MEC-U19 **TOPIC FAMILY: Mechanics** **TOPIC: Center of Mass and Systems of Particles** **COURSE LEVEL: Calculus-based introductory university physics**	**BEST USED** ✓ **To reinforce key concepts** ✓ Before starting homework ✓ Before a quiz or exam

Continue with Physics Sensei →

Physics Sensei is an independent educational resource organized around core college-physics ideas, problem-solving models, and study workflows.

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 60 minutes

1 **Warm-Up Check — page 4** Activate prior knowledge.	2 **Core Concepts — page 6** Review the essential ideas.	3 **Guided Practice — page 8** Apply what you learned.
4 **Confidence Check — page 10** Confirm your understanding.	5 **Summary — page 12** Review the key ideas.	6 **Next Step — page 16** Continue your learning.

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Choose the correct center-of-mass model for a discrete or continuous mass distribution.

State the discrete-particle expression for rCM and the corresponding integral expression for a continuous distribution. Explain what dm represents.

ACTIVITY 2

Recall Activity 2

Connect the time derivative of center-of-mass position to total momentum.

Starting from rCM = (1/M)Σmi ri for constant total mass, state the result obtained by differentiating with respect to time and identify its connection to total momentum.

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Relate the time derivative of total momentum to net external force.

For a constant-mass system, show the chain of relationships connecting dP/dt, net external force, and M ACM.

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let’s rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Center of Mass for Discrete and Continuous Systems

For discrete particles, center of mass is a mass-weighted average of position vectors. For a continuous body, replace the sum with an integral over differential mass elements. Express dm using the appropriate density model, set physically correct limits, and use symmetry before integrating whenever possible.

$$\mathbf{r}_{\mathrm{CM}} = \tfrac{1}{M}\sum_i m_i \mathbf{r}_i \qquad \mathbf{r}_{\mathrm{CM}} = \tfrac{1}{M}\int \mathbf{r}\,dm \qquad M = \int dm$$

EXAMPLE For a rod on $0 \le x \le L$ with linear density $\lambda(x)$ proportional to x, write $dm = \lambda(x)dx$ and evaluate $x_{CM} = [\int x\, dm]/[\int dm]$. Because more mass lies near $x = L$, the result must lie to the right of $L/2$.

SENSEI NOTE Set up dm before integrating. A correct integral with the wrong mass element or limits is still the wrong physical model.

KEY CONCEPT 2

Center-of-Mass Velocity and Total Momentum

For constant total mass, differentiating the center-of-mass position gives the center-of-mass velocity. Multiplying by total mass produces the system’s total momentum. This provides the bridge from particle-level motion to a single system-level velocity.

$$\mathbf{V}_{\mathrm{CM}} = \frac{d\mathbf{r}_{\mathrm{CM}}}{dt} \qquad \mathbf{P} = \sum_i m_i \mathbf{v}_i = M\mathbf{V}_{\mathrm{CM}}$$

EXAMPLE For particles of constant masses, differentiate $r_{CM} = (1/M)\Sigma m_i r_i$ to obtain $V_{CM} = (1/M)\Sigma m_i v_i$, so $M V_{CM} = \Sigma m_i v_i = P$.

SENSEI NOTE The simple relation $P = M V_{CM}$ assumes the system mass is treated consistently. For the introductory mechanics systems here, M is constant.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

External Force, Momentum, and Center-of-Mass Acceleration

Newton's second law for the system follows from the rate of change of total momentum. Internal force pairs cancel from the system force sum under the standard Newtonian model, leaving the net external force. For constant total mass, differentiating P = M VCM gives net external force = M ACM.

$$\frac{d\mathbf{P}}{dt} = \sum \mathbf{F}_{\text{ext}} = M\mathbf{A}_{\text{CM}}$$

EXAMPLE If net external force = 0, then dP/dt = 0, so P is constant. For constant M, VCM is constant as well, even if particles collide, explode apart, or exchange momentum internally.

SENSEI NOTE Internal forces may be large, but they do not appear in the net external-force equation for the complete system. Define the system boundary before summing forces.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Build the differential mass element, then integrate the weighted position.

A thin rod extends from $x = 0$ to $x = L$ with linear density $\lambda(x) = kx$. Find the center of mass in terms of L.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Use the discrete definition, then differentiate to connect center-of-mass motion to momentum.

For a constant-mass particle system, start from $r_{CM} = (1/M)\Sigma m_i r_i$ and derive $P = M V_{CM}$.

PRACTICE 3

Independent Problem

Differentiate total momentum and identify which force terms survive at the system level.

Show that for a constant-mass system dP/dt = net external force leads to A_{CM} = net external force/M, and explain why internal force pairs do not determine A_{CM}.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Set Up a Continuous Center-of-Mass Integral

Choose the correct differential mass element and limits.

A thin rod occupies $0 \le x \le L$ with linear density $\lambda(x) = kx^2$. Write the integral expression for xCM. You do not need to evaluate it.

QUICK CHECK 2

Derive the Momentum Relation

Differentiate the center-of-mass definition for constant total mass.

Starting from rCM = (1/M)Σmi ri, derive the relation between total momentum P and VCM.

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Connect Momentum Rate to External Force

State the system-level chain of relationships for constant total mass.

Show how dP/dt = net external force leads to net external force = M ACM, and explain why internal force pairs do not determine ACM for the complete system.

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Choose Σ or ∫ from the Mass Model

Use a discrete sum for particles and an integral with the correct dm for continuous mass distributions.

KEY TAKEAWAY 2

Differentiate rCM to Reach System Momentum

For constant total mass, differentiating center-of-mass position gives P = M VCM.

KEY TAKEAWAY 3

Differentiate P to Reach External-Force Dynamics

The system relation dP/dt = net external force gives net external force = M ACM for constant total mass.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You’ve reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to identify the step that supports the result.

WARM-UP • ACTIVITY 1

Recall Activity 1

Discrete: $r_{CM} = (1/M)\Sigma m_i r_i$. Continuous: $r_{CM} = (1/M)\int r\, dm$, with $M = \int dm$.

WHY IT WORKS A discrete set contributes separate masses m_i. A continuous body is partitioned into differential mass elements dm, which are expressed from the relevant density model such as $dm = \lambda\, dx$.

WARM-UP • ACTIVITY 2

Recall Activity 2

$V_{CM} = (1/M)\Sigma m_i v_i$, so $P = \Sigma m_i v_i = M V_{CM}$.

WHY IT WORKS For constant M, differentiate $r_{CM} = (1/M)\Sigma m_i r_i$ to obtain $V_{CM} = (1/M)\Sigma m_i v_i$. Multiplying by M gives $M V_{CM} = \Sigma m_i v_i = P$.

WARM-UP • ACTIVITY 3

Recall Activity 3

dP/dt = net external force = $M A_{CM}$.

WHY IT WORKS Newton's second law for the complete system gives dP/dt = net external force. With $P = M V_{CM}$ and constant M, differentiating gives $dP/dt = M A_{CM}$.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

xCM = 2L/3.

WHY IT WORKS dm = kx dx. Then $M = \int_0^L kx\,dx = kL^2/2$ and $\int_0^L x\,dm = \int_0^L kx^2\,dx = kL^3/3$. Therefore xCM = $(kL^3/3)/(kL^2/2) = 2L/3$.

PRACTICE 2

Guided Problem

P = M VCM.

WHY IT WORKS Differentiate rCM = (1/M)Σmi ri for constant masses: VCM = (1/M)Σmi vi. Multiplying by M gives M VCM = Σmi vi, which is the total momentum P.

PRACTICE 3

Independent Problem

ACM = net external force/M for constant total mass; internal force pairs do not determine ACM.

WHY IT WORKS dP/dt = net external force. Since P = M VCM and M is constant, dP/dt = M ACM. Internal Newtonian force pairs cancel from the complete system force sum.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Set Up a Continuous Center-of-Mass Integral

$x_{CM} = [\int_0^L x(kx^2)dx]/[\int_0^L kx^2dx]$.

WHY IT WORKS The mass element is $dm = \lambda(x)dx = kx^2dx$. Substitute dm into $x_{CM} = (1/M)\int x\, dm$, with $M = \int dm$ and both integrals evaluated from 0 to L.

QUICK CHECK 2

Derive the Momentum Relation

$P = M V_{CM}$.

WHY IT WORKS For constant total mass, differentiating the discrete center-of-mass definition gives $V_{CM} = (1/M)\Sigma m_i v_i$. The numerator is total momentum, so $P = M V_{CM}$.

QUICK CHECK 3

Connect Momentum Rate to External Force

dP/dt = net external force = $M A_{CM}$. Internal force pairs can redistribute momentum among particles but do not determine A_{CM} for the complete system.

WHY IT WORKS Newton's second law for the system gives dP/dt = net external force. With constant M and $P = M V_{CM}$, differentiation gives net external force = $M A_{CM}$; internal pair forces cancel in the system sum.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

? **I'm Still Unsure**	◎ **I Need More Practice**	✓ **I'm Ready**
Review the key ideas and examples again.	Continue with additional practice for this topic.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com