FULL REVIEW

Full Review: Friction, Inclines, and Connected Objects — Calculus-Based

Review the essential ideas, relationships, and problem-solving tools for Friction, Inclines, and Connected Objects.

◷ TIME	◎ BEST FOR	✓ FINISH WITH
45–60 minutes	A complete topic review	A readiness check

After this full review, you'll be able to…
recall the essential ideas, apply them to representative problems, and determine what to study next.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

Start the Review Online →
Watch the Video Review →

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • TOPIC ALIGNMENT

Topic Alignment

This bundle is aligned to the approved Physics Sensei topic specification below. Use it to recover the topic structure, reinforce key decisions, and confirm readiness for the next study task.

TEXTBOOK: Independent Physics Sensei Unit Review **CHAPTER: Mechanics • MEC-U17** **TOPIC: Friction, Inclines, and Connected Objects** **COURSE LEVEL: Calculus-Based**	**BEST USED** **✓ After reading the chapter** **✓ Before starting homework** **✓ Before a quiz or exam**

Review Prerequisites →

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

1 **Warm-Up Check — page 4** Activate prior knowledge.	2 **Core Concepts — page 6** Review the essential ideas.	3 **Guided Practice — page 8** Apply what you learned.
4 **Confidence Check — page 10** Confirm your understanding.	5 **Summary — page 12** Review the key ideas.	6 **Next Step — page 16** Continue your learning.

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

Recall Activity 1

Classify the friction model before calculating.

For a contact described by the Coulomb model, state the static-friction condition as an inequality and explain what changes at impending slip.

ACTIVITY 2

Recall Activity 2

Choose incline components and the normal force.

Using axes tangent and normal to a fixed incline, express the gravitational generalized components and explain why the normal equation is algebraic when there is no normal acceleration.

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Identify the shared constraint in a connected system.

If the rope-length constraint is $x_1 + x_2 = L$, differentiate it twice to obtain the velocity and acceleration constraints.

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Friction is a contact response, not a preset force

The Coulomb model is piecewise: the static contact force is a constraint reaction within a friction cone/interval, while sliding selects a kinetic magnitude opposite relative tangential velocity. The transition occurs at the static bound.

$|f_t| \le \mu_s N$ in stick; during slip $f_t = -\mu_k N\,\mathrm{sgn}(v_{rel,t})$ for the simple 1D tangential model.

EXAMPLE As an applied tangential force increases quasistatically, the static reaction follows it until $|f_t|=\mu_s N$; beyond that point the sticking constraint cannot be maintained.

SENSEI NOTE Ask first: is the contact sticking or sliding? Then compute the normal force. Only after those decisions should you choose a friction equation.

KEY CONCEPT 2

Inclines are Newton's laws in rotated axes

Parameterizing the constrained motion by a coordinate s along the incline removes the normal degree of freedom. Newton's second law projected along the tangent supplies the dynamical equation, while the normal projection determines the constraint force N.

$m(d^2s/dt^2) = \Sigma F \cdot \hat{t}$; N from $\Sigma F \cdot \hat{n} = 0$. At impending slip $\tan\theta = \mu_s$ for a block with no other contact forces.

EXAMPLE For a slowly increased incline angle, the sticking inequality $mg \sin\theta \le \mu_s mg \cos\theta$ loses feasibility at $\theta=\arctan \mu_s$.

SENSEI NOTE Draw the weight vector straight down first. Components are mathematical projections of weight, not additional forces.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Connected objects require separate dynamics plus one constraint

Use the rope-length constraint to reduce the number of independent coordinates. Differentiating the constraint gives velocity and acceleration relations; projecting Newton's law or using a generalized coordinate produces the same physical acceleration when signs are consistent.

$C(q_1, q_2) = 0$; d^2C/dt^2 supplies the acceleration constraint. For $x_1 + x_2 = L$, $a_1 + a_2 = 0$.

EXAMPLE Choosing one generalized coordinate x for the same system automatically enforces the rope constraint; the effective inertia is m_1+m_2 and the generalized driving force is $m_2g-\mu_k m_1g$.

SENSEI NOTE Do not write one giant force equation before isolating the objects. Separate free-body diagrams prevent missing friction, weight, or tension terms.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

Worked Example

Test static friction first; if it cannot hold, switch to kinetic friction and solve the motion.

For a block on an incline, use the stick condition to derive the range of θ for which rest is possible in the simple gravity-only case, then state the sliding equation once that condition fails.

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Resolve forces on the incline and keep normal and tangential equations separate.

Show that for constant μ_k on a fixed incline, the acceleration during downward sliding is $a=g(\sin\theta-\mu_k \cos\theta)$, independent of mass.

PRACTICE 3

Independent Problem

Draw both free-body diagrams, define one positive system direction, and solve with the rope constraint.

Using one generalized coordinate x for the same system, derive the equation $(m_1 + m_2)(d^2x/dt^2) = m_2g - \mu_k m_1 g$ and evaluate d^2x/dt^2.

Ready to check your understanding?

You’ve practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you’re ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

Static or kinetic friction check

Answer and justify briefly.

In the stick regime, is f_s determined by $\mu_s N$ alone or by the remaining equations plus the inequality constraint?

QUICK CHECK 2

Incline setup check

Choose the correct component or relation.

At impending downward slip under gravity alone, what relation connects θ and μ_s?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Connected-system check

Identify the valid statement.

If $x_1 + x_2 = L$, what acceleration relation follows?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary — page 12 →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Friction must be classified before it is calculated

Use static friction as an adjustable force satisfying $|f_s| \le \mu_s N$. Use $\mu_s N$ only at impending slip. For sliding, use the kinetic-friction model with direction opposite relative tangential motion.

KEY TAKEAWAY 2

Rotate the axes, not the physics

On a simple incline, resolve weight into $mg \sin\theta$ along the surface and $mg \cos\theta$ perpendicular to it. Determine N from the normal equation, then write the tangential Newton's-law equation.

KEY TAKEAWAY 3

Connected systems combine separate free-body diagrams with one constraint

Write one dynamics equation per object, choose a consistent positive direction, and apply the rope-length acceleration relation. Equal tension requires the ideal single-rope model; equal acceleration does not imply equal net force.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Replace each placeholder with a concise answer and the essential explanation.

WARM-UP • ACTIVITY 1

Recall Activity 1

$|f_s| \le \mu_s N$. At impending slip the magnitude reaches $\mu_s N$; after sliding begins the kinetic-friction model is used.

WHY IT WORKS Static friction supplies only the amount needed to prevent relative slipping, up to its limiting value. Do not set $f_s=\mu_s N$ unless the contact is at impending motion.

WARM-UP • ACTIVITY 2

Recall Activity 2

Q_s=mg sinθ down the slope and the normal balance gives $N-mg\cos\theta=0$ because the holonomic surface constraint fixes the normal coordinate.

WHY IT WORKS For a simple fixed incline, resolving weight into parallel and perpendicular components reduces the force equations to the natural directions of motion and constraint.

WARM-UP • ACTIVITY 3

Recall Activity 3

$v_1 + v_2 = 0$ and $a_1 + a_2 = 0$ for coordinates measured in the same positive sense along the two rope segments.

WHY IT WORKS The rope geometry creates the kinematic constraint. Equal acceleration magnitudes come from constant rope length, not from equal forces.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

Worked Example

Stick requires $mg \sin\theta \le \mu_s mg \cos\theta$, so $\tan\theta \le \mu_s$. Once sliding downward begins, $m(d^2s/dt^2) = mg \sin\theta - \mu_k mg \cos\theta$.

WHY IT WORKS Static friction is a feasibility test: compare the friction required for zero acceleration with the maximum available. Only after failure do you use kinetic friction.

PRACTICE 2

Guided Problem

$m(d^2s/dt^2) = mg \sin\theta - \mu_k mg \cos\theta$; divide by m to obtain $a=g(\sin\theta-\mu_k \cos\theta)$.

WHY IT WORKS Incline problems become straightforward when weight is resolved once, N is obtained from the normal equation, and friction is then placed opposite sliding.

PRACTICE 3

Independent Problem

With the rope constraint built into x, total kinetic energy is $1/2(m_1+m_2)(dx/dt)^2$ and the driving generalized force is $m_2g-\mu_k m_1 g$. Hence $a=11.76/6.0=1.96$ m/s^2.

WHY IT WORKS The two-body equations and the one-coordinate system equation are equivalent. The advantage of separate free-body diagrams is that they also reveal the internal tension.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Static or kinetic friction check

By the remaining equations plus $|f_s| \leq \mu_s N$; $\mu_s N$ is only the bound.

WHY IT WORKS Static friction is a constraint reaction that adjusts to satisfy no-slip dynamics until the limiting value is reached.

QUICK CHECK 2

Incline setup check

$\tan\theta = \mu_s$.

WHY IT WORKS The parallel component drives sliding; the perpendicular component determines the normal force in the simple fixed-incline model.

QUICK CHECK 3

Connected-system check

$a_1 + a_2 = 0$ for coordinates defined in the same positive sense.

WHY IT WORKS The rope sets a kinematic relation; Newton's second law still applies separately to each mass.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

? I'm Still Unsure	◎ I Need More Practice	✓ I'm Ready
Review the key ideas and examples again.	Continue with additional practice for this topic.	Continue to the next recommended resource.
Review again: return to Warm-Up Check on page 4.	**Go to Practice →**	**Continue →**

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

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