FOCUSED REVIEW

# Focused Review: Springs and Elastic Potential Energy

Review the most important ideas, reinforce the essential skills, and confirm you're ready to move on.

| TIME | BEST FOR | FINISH WITH |
|---|---|---|
| 15–20 minutes | Focused review | Greater confidence |

**After this focused review, you'll be able to…**

derive spring work and elastic potential energy from F(x), recover force from U(x), and apply conservation of mechanical energy to ideal spring systems.

## HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

**Start the Review Online →**
**Watch the Video Review →**

## YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 15–20 minutes

FR-02 • COURSE ALIGNMENT

# Course Alignment

Use this independent Physics Sensei Unit Review to reinforce key concepts, prepare for homework, or review before a quiz or exam.

| | |
|---|---|
| **RESOURCE: Physics Sensei Unit Review**<br>**UNIT: MEC-U18**<br>**TOPIC: Springs and Elastic Potential Energy**<br>**COURSE LEVEL: College introductory physics • Calculus-Based** | **BEST USED**<br>✓ **After learning the topic**<br>✓ Before starting homework<br>✓ Before a quiz or exam |

**Explore Physics Sensei Reviews →**

Physics Sensei is an independent educational resource. This Unit Review uses Physics Sensei-owned organization, examples, explanations, and practice.

FR-03 • YOUR REVIEW PLAN

# Your Review Plan

Complete these six focused stages in order. Each stage reinforces your understanding and prepares you for a final confidence check.

**6 stages • Approximately 15–20 minutes**

| | | |
|---|---|---|
| 1<br>**Warm-Up Check — page 4**<br>Refresh key ideas. | 2<br>**Core Concepts — page 6**<br>Reinforce the essentials. | 3<br>**Guided Practice — page 7**<br>Strengthen key skills. |
| 4<br>**Confidence Check — page 8**<br>Confirm your understanding. | 5<br>**Summary — page 10**<br>Remember the essentials. | 6<br>**Next Step — page 14**<br>Choose your next step. |

**PRINTING TIP**

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

# Warm-Up Check

Before you begin, take a minute to reactivate what you already know. This quick warm-up will help you focus on the most important ideas before moving on.

WARM-UP 1

## Variable Spring Force

Recognize position dependence.

*For $F(x) = -200x$ N, what is the force at $x = 0.050$ m?*

WARM-UP 2

## Work Integral

Integrate the spring force.

*Write the integral for work done by a spring moving from $x_i$ to $x_f$.*

FR-04 • WARM-UP CHECK CONTINUED

WARM-UP 3

## Force from Potential

Differentiate U(x).

*If $U(x) = 40x^2$ J, find F(x).*

**Ready to strengthen your understanding?**

You've refreshed the most important ideas. Now reinforce the concepts you'll use most often.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

# Core Concepts

Let's reinforce the most important concepts you'll need to solve problems with confidence. Focus on the ideas that appear most often in homework, quizzes, and exams.

KEY CONCEPT 1

### Position-Dependent Spring Force

The ideal spring force is F(x) = -kx. Because force varies with position, work is an integral. The F-x graph is linear with slope -k.

**Fs(x) = -kx; Ws = integral Fs(x) dx; Us = (1/2)kx^2.**

EXAMPLE For k = 300 N/m from x = 0.12 m to 0.04 m, integrating gives $W_s$ = 1.92 J.

SENSEI NOTE Endpoint force times displacement is not the general spring-work formula.

KEY CONCEPT 2

### Deriving Elastic Potential Energy

For a conservative force, W_c = -ΔU. Integrating -kx gives $U_s(x) = ½kx^2 + C$; choosing U(0)=0 gives C=0.

**Fx = -dU/dx; E = (1/2)mv^2 + (1/2)kx^2 = constant.**

EXAMPLE The triangular area under the linear F-x graph has magnitude $½kx^2$, matching the change in elastic potential energy.

SENSEI NOTE The energy expression is derived from the force law, not an unrelated formula.

**Ready to apply these ideas?**

You've reinforced the essential concepts. Now strengthen your problem-solving skills by working through a few guided examples.

Continue to Guided Practice — page 7 →

FR-06 • STAGE 3 OF 6

# Guided Practice

Now apply the key concepts you've just reviewed. Work through these focused practice activities to reinforce your problem-solving skills before the final confidence check.

PRACTICE 1

## Guided Example

Integrate the spring force.

*For k = 250 N/m, find the work done by the spring from x = 0.10 m to x = 0.*

PRACTICE 2

## Independent Check

Differentiate a potential function.

*Given $U(x)=25x^2$ J, find F(x) and the corresponding k.*

**Ready for a confidence check?**

You've reinforced the essential skills through focused practice. Now confirm your understanding with two short questions.

**Continue to Confidence Check — page 8 →**

FR-07 • STAGE 4 OF 6

# Confidence Check

You've reinforced the key concepts and practiced the essential skills. Now confirm you're ready to move forward by completing these short confidence checks.

CONFIDENCE CHECK 1

## Potential Slope

Infer force direction.

*At a point where dU/dx > 0, what is the sign of F_x?*

CONFIDENCE CHECK 2

## Stable Equilibrium

Use derivatives of U.

*What conditions identify a smooth stable equilibrium?*

HOW DID IT GO?

**I answered ____ of 2 questions correctly.**

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

**You've confirmed your understanding.**

Take one final look at the essential ideas before choosing your next step.

Continue to Summary — page 10 →

FR-08 • STAGE 5 OF 6

# Summary

Before moving on, take one final look at the essential ideas you'll want to remember.

KEY TAKEAWAY 1

### Integrate Variable Force

$W = \int F(x)dx$; for a spring $F=-kx$.

KEY TAKEAWAY 2

### Potential Generates Force

$U_s=\frac{1}{2}kx^2$ and $F_x=-dU/dx$.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

**Ready for your next step?**

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 14 →

ANSWER KEY • CHECK AFTER ATTEMPTING

# Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to check both the setup and the physical interpretation.

WARM-UP • ACTIVITY 1

### Key Ideas

-10.0 N.

WHY IT WORKS F = -kx = -(200)(0.050) = -10.0 N, directed toward equilibrium.

WARM-UP • ACTIVITY 2

### Common Mistakes

$W_s = \int [x_i \text{ to } x\_f] (-kx)\, dx$.

WHY IT WORKS Spring force varies with position, so work is the integral of F(x) over displacement.

WARM-UP • ACTIVITY 3

### Quick Application

F(x) = -80x N.

WHY IT WORKS F = -dU/dx = -80x.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

## Guided Example

1.25 J.

WHY IT WORKS $W_s = \int_{0.1}^{0}(-250x)dx = -125x^2|_{0.1}^{0} = 1.25$ J.

PRACTICE 2

## Independent Check

F(x)=-50x N and k=50 N/m.

WHY IT WORKS F=-dU/dx=-50x; compare with F=-kx.

SOLUTIONS • CONFIDENCE CHECK

CONFIDENCE CHECK 1

### Spring Turning Point

Negative.

WHY IT WORKS  $F_x = -dU/dx$.

CONFIDENCE CHECK 2

### Stable Equilibrium

$dU/dx = 0$ and $d^2U/dx^2 > 0$.

WHY IT WORKS  Force vanishes at an extremum, and a local minimum corresponds to stable equilibrium.

READINESS GUIDE

2 correct: You're ready to continue. • 1 correct: Review the missed idea, then continue. • 0 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

# Next Step: Continue Learning

Great work! You've completed this focused review. Choose the next resource that best matches how confident you feel.

| ? | ◎ | ✓ |
|---|---|---|
| **I'm Still Unsure** | **I Need More Practice** | **I'm Ready** |
| Review the focused review again to strengthen the essential concepts. | Build confidence with additional practice problems. | Continue to the next topic or learning resource. |
| **Review again: return to Warm-Up Check on page 4.** | **Go to Practice →** | **Continue →** |

RECOMMENDED LINKS

**Review again: return to Warm-Up Check on page 4.**

**Find additional practice →**

**Choose another type of physics help →**

Continue reviewing with these companion resources.

**Start the Review Online →**
**Watch the Video Review →**

**PHYSICS HELP THAT MAKES SENSE**

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

**PhysicsSensei.com**