FULL REVIEW

# Full Review: Springs and Elastic Potential Energy

Review the essential ideas, relationships, and problem-solving tools for springs and elastic potential energy using the Calculus-Based treatment.

| ◷ TIME | ◎ BEST FOR | ✓ FINISH WITH |
|---|---|---|
| 45–60 minutes | A complete topic review | A readiness check |

**After this full review, you'll be able to…**

derive spring work and elastic potential energy from F(x), recover force from U(x), and apply conservation of mechanical energy to ideal spring systems.

## HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

**Start the Review Online →**
**Watch the Video Review →**

## YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • COURSE ALIGNMENT

# Course Alignment

Use this independent Physics Sensei Unit Review to reinforce key concepts, prepare for homework, or review before a quiz or exam.

| | |
|---|---|
| **RESOURCE: Physics Sensei Unit Review**<br>**UNIT: MEC-U18**<br>**TOPIC: Springs and Elastic Potential Energy**<br>**COURSE LEVEL: College introductory physics • Calculus-Based** | **BEST USED**<br>✓ **After learning the topic**<br>✓ Before starting homework<br>✓ Before a quiz or exam |

**Explore Physics Sensei Reviews →**

Physics Sensei is an independent educational resource. This Unit Review uses Physics Sensei-owned organization, examples, explanations, and practice.

FR-03 • YOUR REVIEW PLAN

# Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

**6 stages • Approximately 45–60 minutes**



PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

# Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1

## Variable Spring Force

Recognize position dependence.

*For $F(x) = -200x$ N, what is the force at $x = 0.050$ m?*

ACTIVITY 2

## Work Integral

Integrate the spring force.

*Write the integral for work done by a spring moving from $x_i$ to x_f.*

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

## Force from Potential

Differentiate U(x).

*If $U(x) = 40x^2$ J, find F(x).*

### Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts — page 6 →

FR-05 • STAGE 2 OF 6

# Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

## Position-Dependent Spring Force

The ideal spring force is $F(x) = -kx$. Because force varies with position, work is an integral. The F-x graph is linear with slope $-k$.

**Fs(x) = -kx; Ws = integral from xi to xf of Fs(x) dx.**

EXAMPLE For $k = 300$ N/m from $x = 0.12$ m to 0.04 m, integrating gives $W_s = 1.92$ J.

SENSEI NOTE Endpoint force times displacement is not the general spring-work formula.

KEY CONCEPT 2

## Deriving Elastic Potential Energy

For a conservative force, W_c = $-\Delta U$. Integrating $-kx$ gives $U_s(x) = \frac{1}{2}kx^2 + C$; choosing $U(0)=0$ gives $C=0$.

**Ws = integral(-kx dx) = (1/2)kxi^2 - (1/2)kxf^2; Us(x) = (1/2)kx^2.**

EXAMPLE The triangular area under the linear F-x graph has magnitude $\frac{1}{2}kx^2$, matching the change in elastic potential energy.

SENSEI NOTE The energy expression is derived from the force law, not an unrelated formula.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

## Potential Gradients and Mechanical Energy

A conservative force is recovered from the potential via $F_x = -dU/dx$. Equilibrium occurs where $dU/dx = 0$; for an ideal spring $d^2U/dx^2 = k > 0$, so equilibrium is stable.

**Fx = -dU/dx; E = (1/2)mv^2 + (1/2)kx^2 = constant for conservative motion.**

EXAMPLE At total energy E, turning points satisfy $E = U(x)$, and $v(x)=\sqrt{(k/m)(A^2-x^2)}$.

SENSEI NOTE Use calculus where it explains variable force or potential; full SHM differential-equation dynamics belongs to the SHM unit.

### Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice — page 8 →

FR-06 • STAGE 3 OF 6

# Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1

## Worked Example

Integrate the spring force.

*For $k = 250$ N/m, find the work done by the spring from $x = 0.10$ m to $x = 0$.*

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

## Guided Problem

Differentiate a potential function.

*Given $U(x)=25x^2$ J, find $F(x)$ and the corresponding $k$.*

PRACTICE 3

## Independent Problem

Use energy as a function of position.

*A 0.40 kg block on a 160 N/m spring has amplitude 0.15 m. Find speed at $x=0.090$ m.*

### Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check — page 10 →

FR-07 • STAGE 4 OF 6

# Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1

## Potential Slope

Infer force direction.

*At a point where $dU/dx > 0$, what is the sign of $F_x$?*

QUICK CHECK 2

## Stable Equilibrium

Use derivatives of U.

*What conditions identify a smooth stable equilibrium?*

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

## Work and Energy

Connect the integral to potential.

*A spring moves toward equilibrium from $x_i$ to a smaller |x_f|. What are the signs of $W_s$ and $\Delta U_s$?*

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HOW DID IT GO?

**I answered ____ of 3 questions correctly.**

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

**You've checked your understanding.**

Take one final look at the essential ideas before deciding what to do next.

**Continue to Summary — page 12 →**

FR-08 • STAGE 5 OF 6

# Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

### Integrate Variable Force

$W = \int F(x)dx$; for a spring $F=-kx$.

KEY TAKEAWAY 2

### Potential Generates Force

$U_s=\frac{1}{2}kx^2$ and $F_x=-dU/dx$.

KEY TAKEAWAY 3

### Energy Curves Encode Motion

Turning points satisfy $E=U$; stable equilibrium occurs at a local minimum of U.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

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**Ready for your next step?**

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step — page 16 →

ANSWER KEY • CHECK AFTER ATTEMPTING

# Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Use each answer and explanation to identify the step that supports the result.

WARM-UP • ACTIVITY 1

### Recall Activity 1

-10.0 N.

WHY IT WORKS $F = -kx = -(200)(0.050) = -10.0$ N, directed toward equilibrium.

WARM-UP • ACTIVITY 2

### Recall Activity 2

$W_s = \int[x_i \text{ to } x_f] (-kx)\,dx$.

WHY IT WORKS Spring force varies with position, so work is the integral of $F(x)$ over displacement.

WARM-UP • ACTIVITY 3

### Recall Activity 3

$F(x) = -80x$ N.

WHY IT WORKS $F = -dU/dx = -80x$.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1

## Worked Example

1.25 J.

WHY IT WORKS $W_s = \int_{0.1}^{0}(-250x)dx = -125x^2|_{0.1}^{0} = 1.25$ J.

PRACTICE 2

## Guided Problem

$F(x)=-50x$ N and $k=50$ N/m.

WHY IT WORKS $F=-dU/dx=-50x$; compare with $F=-kx$.

PRACTICE 3

## Independent Problem

2.40 m/s.

WHY IT WORKS $v=\sqrt{[(k/m)(A^2-x^2)]} = \sqrt{[400(0.0225-0.0081)]} = 2.40$ m/s.

## SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

### Work and Energy

Negative.

WHY IT WORKS $F_x = -dU/dx$.

QUICK CHECK 2

### Check Spring Energy Reasoning

$dU/dx = 0$ and $d^2U/dx^2 > 0$.

WHY IT WORKS Force vanishes at an extremum, and a local minimum corresponds to stable equilibrium.

QUICK CHECK 3

### Work and Energy

$W_s > 0$ and $\Delta U_s < 0$.

WHY IT WORKS For a conservative spring, $W_s = -\Delta U_s$.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

# Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

| ? I'm Still Unsure | ◎ I Need More Practice | ✓ I'm Ready |
|---|---|---|
| Review the key ideas and examples again. | Continue with additional practice for this topic. | Continue to the next recommended resource. |
| Review again: return to Warm-Up Check on page 4. | Go to Practice → | Continue → |

RECOMMENDED LINKS

Review again: return to Warm-Up Check on page 4.

Find additional practice →

Choose another type of physics help →

Continue reviewing with these companion resources.

Start the Review Online →
Watch the Video Review →

**PHYSICS HELP THAT MAKES SENSE**

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com