

FULL REVIEW

Full Review: Static Equilibrium and Stability

Review the essential ideas, relationships, and problem-solving tools for static equilibrium and stability.

 TIME 45–60 minutes	 BEST FOR A complete topic review	 FINISH WITH A readiness check
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After this full review, you'll be able to...

recall the equilibrium and stability framework, apply it to multi-step rigid-body problems, and evaluate readiness at the calculus-based level.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

[Start the Review Online →](#)

[Watch the Video Review →](#)

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • COURSE ALIGNMENT

Course Alignment

This Physics Sensei Unit Review is an independent learning resource. Use it to reinforce key concepts, prepare for homework, or review before a quiz or exam.

RESOURCE: Physics Sensei Unit Review
UNIT ID: MEC-U23
TOPIC: Static Equilibrium and Stability
COURSE LEVEL: Calculus-Based

BEST USED

- ✓ After learning the unit
- ✓ Before starting homework
- ✓ Before a quiz or exam

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

1 Warm-Up Check Activate prior knowledge.	2 Core Concepts Review the essential ideas.	3 Guided Practice Apply what you learned.
4 Confidence Check Confirm your understanding.	5 Summary Review the key ideas.	6 Next Step Continue your learning.

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1**Vector Conditions**

State the equilibrium equations for a rigid body.

State the equilibrium equations for a rigid body.

ACTIVITY 2**Energy Criterion**

What condition identifies an equilibrium configuration in $U(q)$?

What condition identifies an equilibrium configuration in $U(q)$?

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Quick Application

If $d^2U/dq^2 > 0$ at an equilibrium point, classify it.

If $d^2U/dq^2 < 0$ at an equilibrium point, classify it.

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Vector Force and Torque Balance

Rigid-body equilibrium requires both translational and rotational equations. In three dimensions, the vector equations supply up to six scalar conditions. Torque about an origin is the sum of $\mathbf{r} \times \mathbf{F}$ for all external forces.

$$\Sigma \mathbf{F} = 0 \quad \bullet \quad \Sigma \boldsymbol{\tau} = 0 \quad \bullet \quad dU/dq = 0$$

EXAMPLE A force whose line of action passes through the chosen origin contributes zero torque about that origin.

SENSEI NOTE Changing the origin changes individual torques, but for an equilibrium system the total torque remains zero.

KEY CONCEPT 2

Potential-Energy Stability

For a conservative one-coordinate system, equilibrium occurs at $dU/dq = 0$. A positive second derivative indicates a local minimum and stable equilibrium; a negative second derivative indicates a local maximum and unstable equilibrium.

$$\Sigma \mathbf{F} = 0 \quad \bullet \quad \Sigma \boldsymbol{\tau} = 0 \quad \bullet \quad dU/dq = 0$$

EXAMPLE Near a stable equilibrium q_0 , U can be approximated by a quadratic minimum.

SENSEI NOTE If the second derivative is zero, higher-order terms may be needed to classify the equilibrium.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Small-Angle Stability and Restoring Torque

Near a stable equilibrium, expand U about q_0 . If the leading nonzero curvature is positive, the restoring generalized force is approximately proportional to $-(q - q_0)$. This connects static stability to small oscillations.

$$\Sigma \mathbf{F} = 0 \quad \cdot \quad \Sigma \tau = 0 \quad \cdot \quad dU/dq = 0$$

EXAMPLE With $U \approx U_0 + \frac{1}{2}k(q - q_0)^2$, the generalized restoring force is approximately $-k(q - q_0)$.

SENSEI NOTE A positive curvature gives a restoring response; a negative curvature drives the system away.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

Continue to Guided Practice →

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1**Worked Example**

Solve the problem systematically and show the key equilibrium steps.

A horizontal beam of length L and weight W is hinged at one end and supported by a cable with tension T at angle θ to the beam at the far end. Derive T .

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Solve the problem systematically and show the key equilibrium steps.

A system has $U(q) = aq^2 + bq^4$ with $a > 0$ and $b > 0$. Classify $q = 0$.

PRACTICE 3

Independent Problem

Solve the problem systematically and show the key equilibrium steps.

For $U(\theta) = U_0 + A(1 - \cos \theta)$, with $A > 0$, classify $\theta = 0$ and $\theta = \pi$.

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1**Vector Torque**

Evaluate the direction.

If r points in $+x$ and F points in $+y$, what is the direction of $\tau = r \times F$?

QUICK CHECK 2**Stability from Curvature**

Classify the point.

At $q = q_0$, $dU/dq = 0$ and $d^2U/dq^2 < 0$. What type of equilibrium is present?

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Small-Displacement Model

Interpret the expansion.

If $U(q) \approx U_0 + \frac{1}{2}k(q - q_0)^2$ with $k > 0$, what is the approximate generalized force near q_0 ?

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1

Equilibrium is vector balance

Both ΣF and $\Sigma \tau$ must vanish for a rigid body in static equilibrium.

KEY TAKEAWAY 2

Stability can be read from energy

A local minimum of potential energy corresponds to stable equilibrium in conservative systems.

KEY TAKEAWAY 3

Curvature controls local response

Positive energy curvature produces a restoring tendency and connects stable equilibrium to oscillatory motion.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Focus on the method and the physical meaning of each result.

WARM-UP • ACTIVITY 1

Vector Conditions

$\Sigma \mathbf{F} = 0$ and $\Sigma \boldsymbol{\tau} = 0$, with $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$.

WHY IT WORKS The answer follows from the defining conditions for equilibrium and stability.

WARM-UP • ACTIVITY 2

Energy Criterion

$dU/dq = 0$ for the generalized coordinate q .

WHY IT WORKS The answer follows from the defining conditions for equilibrium and stability.

WARM-UP • ACTIVITY 3

Quick Application

Stable; the potential energy has a local minimum.

WHY IT WORKS The answer follows from the defining conditions for equilibrium and stability.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1**Worked Example**

Torque balance about the hinge gives $TL \sin \theta - W(L/2) = 0$, so $T = W/(2 \sin \theta)$.

WHY IT WORKS A correct solution satisfies both the force and torque requirements, or the stated stability criterion.

PRACTICE 2**Guided Problem**

$dU/dq = 2aq + 4bq^3$, so $q = 0$ is an equilibrium. $d^2U/dq^2 = 2a + 12bq^2$, which is positive at $q = 0$, so the equilibrium is stable.

WHY IT WORKS A correct solution satisfies both the force and torque requirements, or the stated stability criterion.

PRACTICE 3**Independent Problem**

$dU/d\theta = A \sin \theta$, so both are equilibria. $d^2U/d\theta^2 = A \cos \theta$: positive at 0, so stable; negative at π , so unstable.

WHY IT WORKS A correct solution satisfies both the force and torque requirements, or the stated stability criterion.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Vector Torque

+z by the right-hand rule.

WHY IT WORKS The result follows from the equilibrium equations, torque geometry, or energy-stability test.

QUICK CHECK 2

Stability from Curvature

Unstable equilibrium; U has a local maximum.

WHY IT WORKS The result follows from the equilibrium equations, torque geometry, or energy-stability test.

QUICK CHECK 3

Small-Displacement Model

$Q = -dU/dq \approx -k(q - q_0)$, a restoring force toward q_0 .

WHY IT WORKS The result follows from the equilibrium equations, torque geometry, or energy-stability test.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

 I'm Still Unsure Review the key ideas and examples again. Review Again →	 I Need More Practice Continue with additional practice for this topic. Go to Practice →	 I'm Ready Continue to the next recommended resource. Continue →
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RECOMMENDED LINKS

[Review this topic again →](#)[Find additional practice →](#)[Choose another type of physics help →](#)

Continue reviewing with these companion resources.

[Start the Review Online →](#)[Watch the Video Review →](#)

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com