

FULL REVIEW

Full Review — Vector Components and Unit Vectors — Calculus-Based

Review the essential ideas, relationships, and problem-solving tools for Vector Components and Unit Vectors.

 TIME 45–60 minutes	 BEST FOR A complete topic review	 FINISH WITH A readiness check
---	---	--

After this full review, you'll be able to...

recall the essential ideas, apply them to representative problems, and determine what to study next.

HOW TO USE THIS GUIDE

Work through the stages in order. Write directly in the spaces provided, check your work in the Solutions section, and finish by choosing the next resource that matches your confidence.

Choose how you want to review: work through this guide, follow the online review, or watch the video review.

[Start the Review Online →](#)

[Watch the Video Review →](#)

YOU WILL NEED

Pencil or pen • Calculator if needed • Approximately 45–60 minutes

FR-02 • COURSE ALIGNMENT

Course Alignment

This Physics Sensei Topic Review is an independent learning resource. Use it to reinforce key concepts, prepare for homework, or review before a quiz or exam.

Related Unit Review: If you need to review the complete unit material, review [Vectors and Components here →](#)

RESOURCE: Physics Sensei Topic Review
TOPIC ID: MEC-U02-T01
TOPIC: Vector Components and Unit Vectors
PARENT UNIT: MEC-U02 — Vectors and Components
COURSE LEVEL: Calculus-Based

BEST USED

- ✓ After learning the topic
- ✓ Before starting homework
- ✓ Before a quiz or exam

FR-03 • YOUR REVIEW PLAN

Your Review Plan

Complete these six stages in order. Each stage builds on the previous one and prepares you for the final readiness check.

6 stages • Approximately 45–60 minutes

1 Warm-Up Check Activate prior knowledge.	2 Core Concepts Review the essential ideas.	3 Guided Practice Apply what you learned.
4 Confidence Check Confirm your understanding.	5 Summary Review the key ideas.	6 Next Step Continue your learning.

PRINTING TIP

For student copies, print double-sided and keep the Solutions section after the student activities so learners attempt each activity before checking their work.

FR-04 • STAGE 1 OF 6

Warm-Up Check

Before you begin, take a moment to see what you already remember. Do not worry about getting everything right. This is only a starting point.

ACTIVITY 1**Recall Activity 1**

Identify coefficients in the Cartesian basis.

For $A = 3 \hat{i} - 4 \hat{j} + 12 \hat{k}$, list A_x , A_y , and A_z .

ACTIVITY 2**Recall Activity 2**

Connect components to differentiation.

If $\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$, what are the components of $d\mathbf{r}/dt$ in a fixed Cartesian basis?

FR-04 • WARM-UP CHECK CONTINUED

ACTIVITY 3

Recall Activity 3

Check a unit vector.

Given $A=(3,4,0)$, write the unit vector $\hat{A}$ in the direction of A .

Ready to strengthen your understanding?

You've refreshed what you already know. Next, reinforce the essential concepts that will help you solve problems with confidence.

Continue to Core Concepts →

FR-05 • STAGE 2 OF 6

Core Concepts

Let's rebuild the key ideas one step at a time. Focus on understanding the relationships before worrying about solving problems.

KEY CONCEPT 1

Components in an Orthonormal Cartesian Basis

The basis $\{\hat{i}, \hat{j}, \hat{k}\}$ is orthonormal, so Cartesian components are projections onto mutually perpendicular unit directions.

$$\mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

EXAMPLE $\mathbf{A}=(1,-2,2)$ has $|\mathbf{A}|=3$.

SENSEI NOTE Component values are basis-dependent; invariant quantities such as magnitude do not depend on a simple rotation of axes.

KEY CONCEPT 2

Direction Unit Vector

Normalize a vector to separate its direction from its magnitude. Then $\mathbf{A}=|\mathbf{A}| \hat{\mathbf{A}}$.

$$\hat{\mathbf{A}}=\mathbf{A}/|\mathbf{A}|, \text{ for } \mathbf{A} \neq \mathbf{0}$$

EXAMPLE For $\mathbf{A}=(2,1,2)$, $\hat{\mathbf{A}}=(2/3, 1/3, 2/3)$.

SENSEI NOTE The zero vector has no defined direction unit vector.

FR-05 • CORE CONCEPTS CONTINUED

KEY CONCEPT 3

Vector-Valued Functions in Fixed Cartesian Coordinates

For a fixed Cartesian basis, vector calculus proceeds componentwise because the basis vectors are constant.

$$\frac{d\mathbf{A}}{dt} = \left(\frac{dA_x}{dt}\right)\hat{i} + \left(\frac{dA_y}{dt}\right)\hat{j} + \left(\frac{dA_z}{dt}\right)\hat{k}$$

EXAMPLE $\mathbf{r}(t) = t^2 \hat{i} + \sin(t) \hat{j}$ gives $\mathbf{v} = 2t \hat{i} + \cos(t) \hat{j}$.

SENSEI NOTE This simple derivative rule changes in coordinate systems whose basis vectors vary with position.

Ready to apply these ideas?

You've reinforced the essential concepts. Now put them into practice by working through guided examples and building your problem-solving confidence.

[Continue to Guided Practice →](#)

FR-06 • STAGE 3 OF 6

Guided Practice

Now it's time to apply what you've reviewed. Work through each activity in order. The examples become gradually more challenging, and each prepares you for the final readiness check.

PRACTICE 1**Worked Example**

For $A = (4, -4, 7)$, find magnitude and direction unit vector.

$$|A| = 9; \hat{A} = (4/9)\hat{i} - (4/9)\hat{j} + (7/9)\hat{k}.$$

FR-06 • GUIDED PRACTICE CONTINUED

PRACTICE 2

Guided Problem

Given $\mathbf{r}(t) = t \hat{i} + t^2 \hat{j} + t^3 \hat{k}$, find $\mathbf{v}(t)$ and $\mathbf{a}(t)$.

$$\mathbf{v} = \hat{i} + 2t \hat{j} + 3t^2 \hat{k}; \quad \mathbf{a} = 2 \hat{j} + 6t \hat{k}.$$

PRACTICE 3

Independent Problem

For $\mathbf{A} = (1, 2, 2)$ and $\mathbf{B} = (-2, 0, 1)$, find $\mathbf{A} + \mathbf{B}$ and its magnitude.

$$\mathbf{A} + \mathbf{B} = (-1, 2, 3); \quad |\mathbf{A} + \mathbf{B}| = \sqrt{14}.$$

Ready to check your understanding?

You've practiced the essential skills with guidance. Now solve a few short problems on your own and confirm you're ready to move forward.

Continue to Confidence Check →

FR-07 • STAGE 4 OF 6

Confidence Check

You've rebuilt the key ideas and practiced them with guidance. Now try these short questions on your own to check your understanding before moving on.

QUICK CHECK 1**Three-Dimensional Magnitude**

Use all Cartesian components.

Find $|A|$ for $A=(6,2,-3)$.

QUICK CHECK 2**Unit Direction**

Normalize the vector.

Find $\hat{A}$ for $A=(0,8,6)$.

FR-07 • CONFIDENCE CHECK CONTINUED

QUICK CHECK 3

Componentwise Derivative

Differentiate each coefficient.

If $A(t) = t^3 \hat{i} + e^t \hat{j}$, find dA/dt .

HOW DID IT GO?

I answered ____ of 3 questions correctly.

Use the Solutions section to check your reasoning, then take one final look at the essential ideas.

You've checked your understanding.

Take one final look at the essential ideas before deciding what to do next.

Continue to Summary →

FR-08 • STAGE 5 OF 6

Summary

Before moving on, take one final look at the most important ideas from this review.

KEY TAKEAWAY 1**Expand**

Represent a Cartesian vector as scalar components multiplying orthonormal basis unit vectors.

KEY TAKEAWAY 2**Normalize**

Use $A/|A|$ to extract direction from any nonzero vector.

KEY TAKEAWAY 3**Differentiate**

In a fixed Cartesian basis, differentiate vector-valued functions component by component.

MY ONE-SENTENCE SUMMARY

In my own words, the most important idea is:

Ready for your next step?

You've reviewed the essential ideas one last time. Now choose the resource that best matches how confident you feel.

Continue to Next Step →

ANSWER KEY • CHECK AFTER ATTEMPTING

Solutions and Explanations

Use these solutions to check your reasoning—not only your final answer. Replace each placeholder with a concise answer and the essential explanation.

WARM-UP • ACTIVITY 1

Recall Activity 1

$A_x=3$, $A_y=-4$, $A_z=12$.

WHY IT WORKS The scalar coefficients of the basis vectors are the Cartesian components.

WARM-UP • ACTIVITY 2

Recall Activity 2

dx/dt and dy/dt .

WHY IT WORKS The Cartesian basis vectors are constant, so differentiation acts on the scalar component functions.

WARM-UP • ACTIVITY 3

Recall Activity 3

$\hat{A}=(3/5)\hat{i}+(4/5)\hat{j}$.

WHY IT WORKS Dividing a vector by its magnitude produces a vector of magnitude 1.

SOLUTIONS • GUIDED PRACTICE

PRACTICE 1**Worked Example**

$$|A|=9; \hat{A}=(4/9)\hat{i}-(4/9)\hat{j}+(7/9)\hat{k}.$$

WHY IT WORKS The result follows from the component equations, magnitude relation, and unit-vector basis representation.

PRACTICE 2**Guided Problem**

$$\mathbf{v}=\hat{i}+2t\hat{j}+3t^2\hat{k}; \mathbf{a}=2\hat{j}+6t\hat{k}.$$

WHY IT WORKS The result follows from the component equations, magnitude relation, and unit-vector basis representation.

PRACTICE 3**Independent Problem**

$$\mathbf{A}+\mathbf{B}=(-1,2,3); |\mathbf{A}+\mathbf{B}|=\sqrt{14}.$$

WHY IT WORKS The result follows from the component equations, magnitude relation, and unit-vector basis representation.

SOLUTIONS • CONFIDENCE CHECK

QUICK CHECK 1

Three-Dimensional Magnitude

7.

WHY IT WORKS The stated component or basis rule gives the result directly.

QUICK CHECK 2

Unit Direction $0\hat{i} + 0.8\hat{j} + 0.6\hat{k}$.

WHY IT WORKS The stated component or basis rule gives the result directly.

QUICK CHECK 3

Componentwise Derivative $3t^2\hat{i} + e^t\hat{j}$.

WHY IT WORKS The stated component or basis rule gives the result directly.

READINESS GUIDE

3 correct: You're ready to continue. • 2 correct: Review the missed idea, then continue. • 0–1 correct: Revisit Core Concepts or choose more practice.

FR-09 • STAGE 6 OF 6

Next Step: Continue Learning

Great work! You've completed this review. Choose the next resource that best matches how confident you feel.

 I'm Still Unsure Review the key ideas and examples again. Review Again →	 I Need More Practice Continue with additional practice for this topic. Go to Practice →	 I'm Ready Continue to the next recommended resource. Continue →
--	---	---

RECOMMENDED LINKS

[Review this topic again →](#)[Find additional practice →](#)[Choose another type of physics help →](#)

Continue reviewing with these companion resources.

[Start the Review Online →](#)[Watch the Video Review →](#)

PHYSICS HELP THAT MAKES SENSE

A Physics Sensei review never ends at the last question. It always guides you toward the next useful step.

PhysicsSensei.com